EBA 1180, lecture 22, 6 Nov. 2024, Runar Ile

Plan 1. Marginal cost, revenue, profit, ...
2. Average unit cost and cost optimum.

1. Marginal cost, revenue, profit, ...

Intro: Diamonds and water

Ex: Cost of removing $x\%$ of pollution from a lake.

$C(x)$ = is the total cost of producing x units
(of some commodity, or...)

$C'(x)$ is (by definition) the marginal cost at x.

Interpretation The cost of producing one more unit than x units

$$= C(x+1) - C(x) = \frac{C(x+1) - C(x)}{1} \approx \lim_{h \to 0} \frac{C(x+h) - C(x)}{h} = C'(x)$$

Why $C'(x)$? - much simpler math to work with!

$R(x)$ = the total revenue of selling x units

$R'(x)$ = the marginal revenue at x.

Ex x = tons of salmon produced and sold

$R'(50) \approx$ extra revenue from selling 51 tons instead of 50 tons
$= R(51) - R(50)$

The profit function: $P(x) = R(x) - C(x)$ $(= \Pi(x)$ - the economist$)$

$P'(x) = R'(x) - C'(x)$

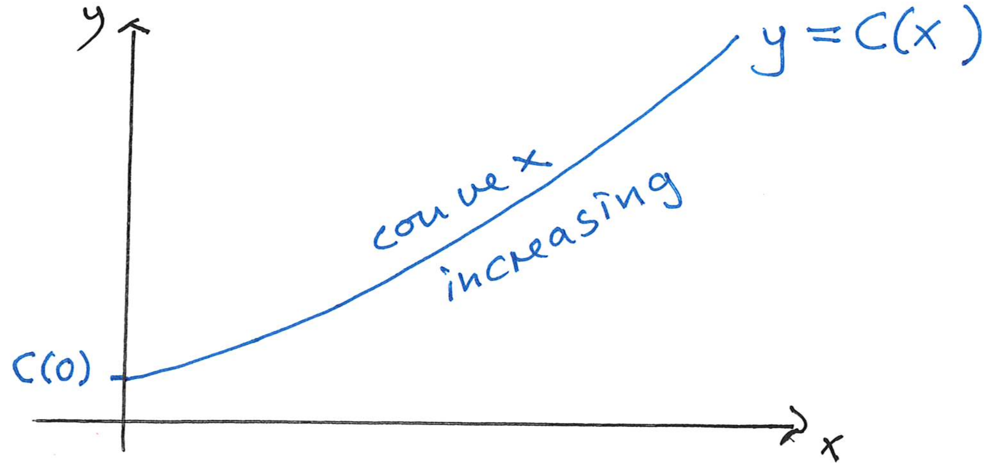
is the marginal profit function.

2. Average unit cost and cost optimum

Average unit cost of producing x units is $A(x) = \dfrac{C(x)}{x}$ – not a constant function!

Definition $C(x)$ is a cost function if

1. $C(0) > 0$ (start-up cost)
2. $C(x)$ is increasing ($C'(x) \geq 0$)
3. $C(x)$ is convex ($C''(x) \geq 0$)



Definition If $x = c$ is the minimum point for $A(x)$, then c is called the cost optimum (the x-value that gives the minimal average unit cost)

Result If $C(x)$ is a cost function with $C''(x) > 0$ for $x > 0$, then there is a cost optimum, and it is the solution of the equation

$$C'(x) = A(x)$$

Start: 11.00

Ex $C(x) = x^2 + 200x + 160\,000$

This is a cost function because:

1. $C(0) = 160\,000 > 0$
2. $C'(x) = 2x + 200 > 0$ for $x \geq 0$
3. $C''(x) = 2 > 0$ for all x

By the result the cost optim exists and is the solution of the equation

$$C'(x) = A(x)$$

$$2x + 200 = \frac{x^2 + 200x + 160\,000}{x}$$

$$\cancel{2}x + \cancel{200} = \cancel{x} + \cancel{200} + \frac{160\,000}{x}$$

$$x = \frac{160\,000}{x} \quad \Big| \cdot x$$

$$x^2 = 160\,000$$

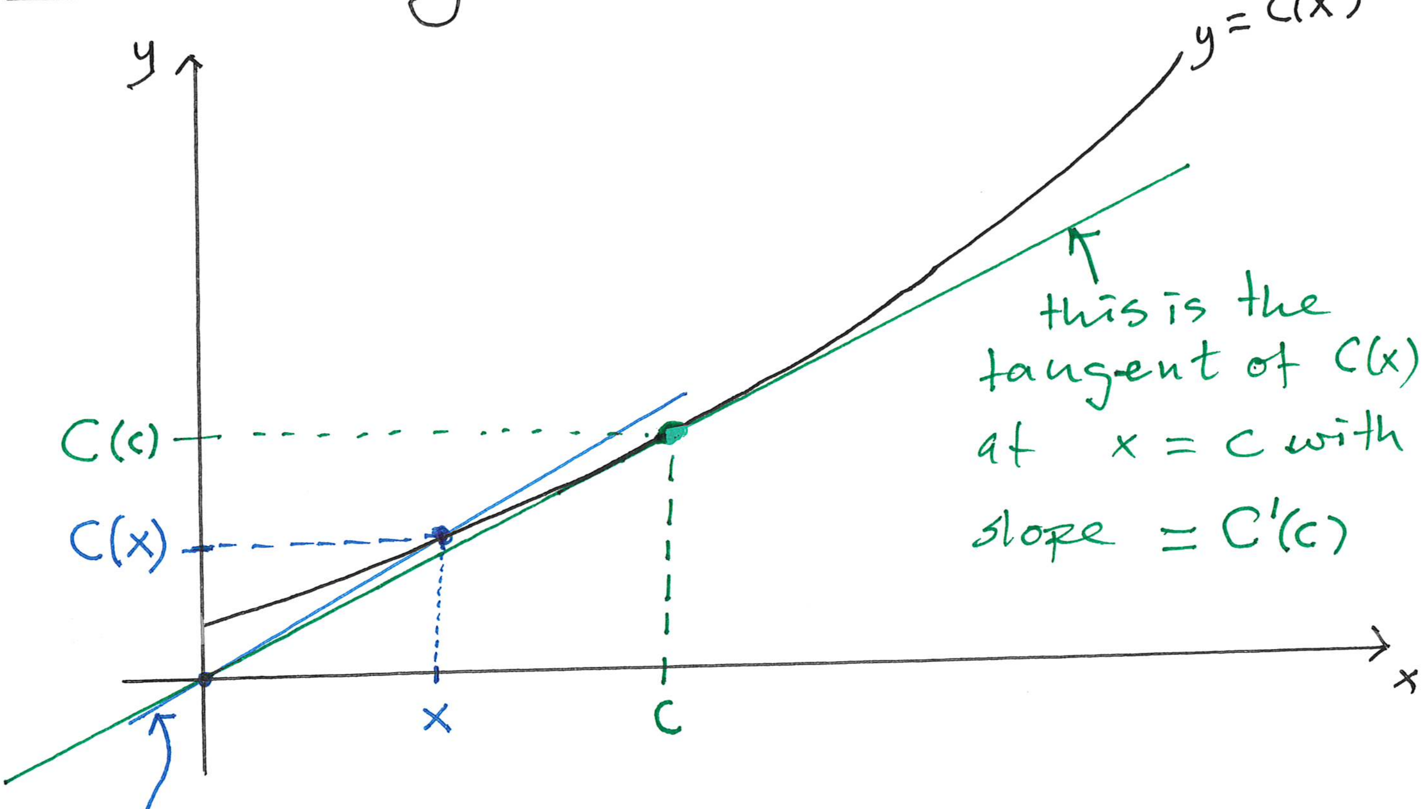
so $\underline{\underline{x = 400}}$ (only positive x)

is the cost optimum (by the result).

The minimal average unit cost is

$$A(400) = C'(400) = 2 \cdot 400 + 200 = \underline{\underline{1000}}$$

Geometric argument for the result



the slope is

$$\frac{C(x)}{x} = A(x) \quad \text{and} \quad A(c) = \frac{C(c)}{c} \quad \text{is}$$

the minimal unit cost when

$C'(c) = A(c)$ = the smallest slope through the origin
= the slope of the <u>tangent</u> of $C(x)$ which goes through the origin (only one!)

Algebraic reason for the result

We determine the stationary point of $A(x)$.

Calculate
$$A'(x) = \left[\frac{C(x)}{x}\right]' \underset{\text{frac. rule}}{=} \frac{C'(x)\cdot x - C(x)\cdot 1}{x^2} \quad \left| \frac{:x}{:x} \right.$$

$$= \frac{C'(x) - A(x)}{x}$$

So $A'(x) = 0$ is equivalent to $C'(x) \overset{(*)}{=} A(x)$.

Assume $x = c$ is such a stationary point, i.e. a solution of $(*)$. We use the second derivative test:

If $A''(c) > 0$ the c is a (loc.) min. point.

Calculate:
$$A''(x) \underset{\text{frac. rule}}{=} \frac{[C''(x) - A'(x)]\cdot x - [C'(x) - A(x)]\cdot 1}{x^2}$$

Substitute $x = C$:
$$A''(c) = \frac{[C''(c) - \overbrace{A'(c)}^{=0}]\cdot c - \overbrace{[C'(c) - A(c)]}^{=0}}{c^2}$$

$$= \frac{C''(c)}{c} > 0 \quad (\text{for } c > 0)$$