EBA 1180, lecture 23, 12 Nov. 2024, Runar Ile

Plan 1. Repetition: l'Hôpital's rule (probl. 1h)
Cost functions (probl. 3c)
2. Elasticity

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1. Repetition l'Hôpital's rule: For limits $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$

- Differentiate the numerator and the denominator seperately.
- Consider the the same limit of the new fraction.

Probl 1h
$$\lim_{x\to 1} \frac{\ln(x)}{\sqrt{x}-1} \underset{"\frac{0}{0}"}{\overset{\text{l'Hôp}}{=}} \lim_{x\to 1} \frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{2\sqrt{x}}\right)} = \frac{\left(\frac{1}{1}\right)}{\left(\frac{1}{2\sqrt{1}}\right)} = \frac{1}{\left(\frac{1}{2}\right)} = \underline{\underline{2}}$$

Meaning: $\frac{\ln(x)}{\sqrt{x}-1}$ has $\underline{\underline{\text{no}}}$ vertical asymptote for $x=1$.

Also (extra!):
$$\lim_{x\to\infty} \frac{\ln(x)}{\sqrt{x}-1} \underset{"\frac{\infty}{\infty}"}{\overset{\text{l'Hôp}}{=}} \lim_{x\to\infty} \frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{2\sqrt{x}}\right)} \quad \Big| \cdot \frac{x\cdot 2\sqrt{x}}{x\cdot 2\sqrt{x}} = 1$$

$$= \lim_{x\to\infty} \frac{2\sqrt{x}}{x} = \lim_{x\to\infty} \frac{2\cancel{\sqrt{x}}}{\sqrt{x}\cdot\cancel{\sqrt{x}}} = \lim_{x\to\infty} \frac{2}{\sqrt{x}} = \underline{\underline{0}}$$

Meaning: $\frac{\ln(x)}{\sqrt{x}-1}$ has horizontal asymptote $y=0$

Cost functions

Probl 3c $C(x) = 400\cdot e^{0.001x^2}$ $(x \geq 0)$

is a cost function because

① $C(0) = 400\cdot e^{0.001\cdot 0^2} = 400\cdot e^0 = 400 > 0$

② $C'(x) \overset{\text{chain rule}}{=} 400\cdot 0.001\cdot 2x\cdot e^{0.001\cdot x^2}$
$$= 0.8\cdot x\cdot e^{0.001\cdot x^2} \geq 0 \text{ for } x \geq 0$$

(3) $C''(x) \overset{\substack{\text{prod. rule} \\ \text{+ chain rule}}}{=} 0.8 \cdot e^{0.001 \cdot x^2} + 0.8x \cdot 0.002 \cdot x \cdot e^{0.001 \cdot x^2}$

$$= \underbrace{0.8}_{>0} \cdot \underbrace{(1 + 0.002\,x^2)}_{>0} \cdot \underbrace{e^{0.001 \cdot x^2}}_{>0} > 0 \quad \text{for all } x$$

Because $C''(x) > 0$, the cost optimum is the solution of the eq.

$$C'(x) \overset{\substack{\text{nice} \\ \text{result}}}{=} A(x) \qquad \left(A(x) = \frac{C(x)}{x}\right)$$

that is $\quad 0.8\,x\,e^{0.001 \cdot x^2} = \dfrac{400\,e^{0.001 \cdot x^2}}{x} \quad \Big| \cdot x$

gives $\quad 0.8\,x^2 e^{0.001x^2} = 400 \cdot e^{0.001 \cdot x^2} \quad \Big| : e^{0.001 \cdot x^2}$

so $\quad 0.8 \cdot x^2 = 400 \quad \Big| : 0.8$

so $\quad x^2 = \dfrac{400}{0.8} = 500$

so $\quad x = \underline{\underline{\sqrt{500}}} = \underline{\underline{22.36}} \qquad (x \geq 0)$

is the cost optimum

The minimal average unit cost is

$$A(\sqrt{500}) \overset{\substack{\text{by the} \\ \text{nice} \\ \text{result}}}{=} C'(\sqrt{500}) = 0.8 \cdot \sqrt{500} \cdot e^{0.001 \cdot (\sqrt{500})^2}$$

$$= \underline{\underline{29.49}}$$

2. Elasticity

p = price/unit

$D(p)$ = demand of commodity with price p

= # units sold if price is p.

The problem of comparing units.

Ex A barrel of North Sea crude oil costs \$ 71.95

A litre of —— " —————— costs NOK 5.01

The price elasticity of the demand is

$$\varepsilon = \frac{\text{relative change in demand}}{\text{relative change in price}}$$

these numbers are independent of the units used

Ex In a month the price of a commodity drops from 12 thousand to 10 thousand and the demand increases from 50 mill. to 60 mill. Then

Start: 11.05

$$\varepsilon = \frac{\left(\frac{60-50}{50}\right)}{\left(\frac{10-12}{12}\right)} = \frac{\left(\frac{10}{50}\right)}{\left(\frac{-2}{12}\right)} = \frac{120}{-100} = \underline{\underline{-1.2}}$$

Interpretation If the price increases by 1 % from 12 000, then the demand decreases by 1.2 %

Theory Assume we have a demand function $D(p)$. If the price is changed from p to $p+h$, the relative change in price is

$$\frac{p+h-p}{p} = \frac{h}{p}.$$

Then

$$\frac{\text{relative change in demand}}{\text{relative change in price}}$$

$$= \frac{\left(\frac{D(p+h)-D(p)}{D(p)}\right)}{\left(\frac{h}{p}\right)} \qquad \Big| \cdot \frac{p\cdot D(p)}{p \cdot D(p)} = 1$$

$$= \frac{D(p+h)-D(p)}{h} \cdot \frac{p}{D(p)}$$

$\big\downarrow\ h \to 0$ (the price change approaches 0)

$$\mathcal{E}(p) = D'(p) \cdot \frac{p}{D(p)} = \frac{D'(p)\cdot p}{D(p)}$$

This is the momentary price elasticity of the demand function.

Ex $D(p) = 50 - p$ for $0 < p < 50$.

Then $D'(p) = -1$ and $\varepsilon(p) = \frac{(-1)\cdot p}{50-p} = \frac{-p}{50-p}$

Important question

Is the revenue going up or down if we increase the price a little?

Revenue $R(p) = p \cdot D(p)$

– is $R(p)$ increasing or decreasing?

$$R'(p) \overset{\text{prod. rule}}{=} 1 \cdot D(p) + p \cdot D'(p)$$

$$= D(p) \cdot \left[1 + \frac{p \cdot D'(p)}{D(p)} \right]$$

$$= \underbrace{D(p)}_{\text{always pos.}} \cdot \underbrace{\left[1 + \varepsilon(p) \right]}_{\text{pos. or neg.?}}$$

If $\varepsilon(p) < -1$ we get neg. $R'(p)$ so $R(p)$ is decreasing

Say: elastic demand

If $\varepsilon(p) > -1$ we get pos. $R'(p)$ so $R(p)$ is increasing

Say: inelastic demand

If $\varepsilon(p) = -1$

then $R'(p) = 0$ and $R(p)$ has a stationary point.

Say: the demand is unit elastic

Ex $D(p) = 50 - p$ $(0 < p < 50)$. We got

$$\varepsilon(p) = \frac{-p}{50-p}$$

Q: In what price range do we have elastic demand?

A: Solve the inequality $\varepsilon(p) < -1$

so $\frac{-p}{50-p} < -1$ $\quad | +1$

get $\frac{-p}{50-p} + 1 < 0$

so $\frac{-p+50-p}{50-p} < 0$

so $\frac{50-2p}{\underbrace{(50-p)}_{\text{pos. since } p<50}} < 0$

so $50 - 2p < 0$, so $50 < 2p$

and get $\underline{p > 25}$

Elastic demand w.r.t. price for $p \in \langle 25, 50 \rangle$

Inelastic ——— " ——— $p \in \langle 0, 25 \rangle$

Unit elastic ——— " ——— $p = 25$