EBA 1180, lecture 24, 13 Nov. 2024, Runar Ile

Plan
1. Repetition: Elasticity
2. Linear approximation (= the tangent line)
3. Higher degree Taylor polynomials
4. About the exam
5. How to prepare for the exam.

1. Rep: Elasticity

p = price/unit, demand function $D(p)$

Ex $D(p) = 200 \cdot e^{-0.01p}$

Calculate $\varepsilon(p)$ — the elasticity function.

We have $D'(p) \overset{\text{chain rule}}{=} -0.01 \cdot 200 \cdot e^{-0.01p} = -2e^{-0.01p}$

So $$\varepsilon(p) = \frac{D'(p) \cdot p}{D(p)} = \frac{-2\cancel{e^{-0.01p}} \cdot p}{200 \cdot \cancel{e^{-0.01p}}} = \underline{\underline{-0.01p}}$$

The demand is elastic w.r.t. price if

$\varepsilon(p) < -1$, that is $-0.01p < -1$ (an inequality!)

that is: $\underline{p > 100}$

Meaning If $p > 100$, then a small increase in the price gives a decline in revenue.

Ex $\varepsilon(110) = -1.1$, so a price increase of 1% from 110 gives a demand decline of 1.1%.

The demand is inelastic w.r.t. price if

$\varepsilon(p) > -1$, that is $-0.01p > -1$ so

$\underline{p < 100}$

__Meaning__ If $p < 100$, a small price increase gives an increase in revenue.

__Ex__ $\varepsilon(80) = -0.8$, so 1% price increase from 80 gives 0.8% demand decrease.

If $\varepsilon(p) = -1$ (so $p = 100$) then demand is __unit elastic__ w.r.t. price.

Meaning: No (very little) change in revenue if price is changed a little from $p = 100$.

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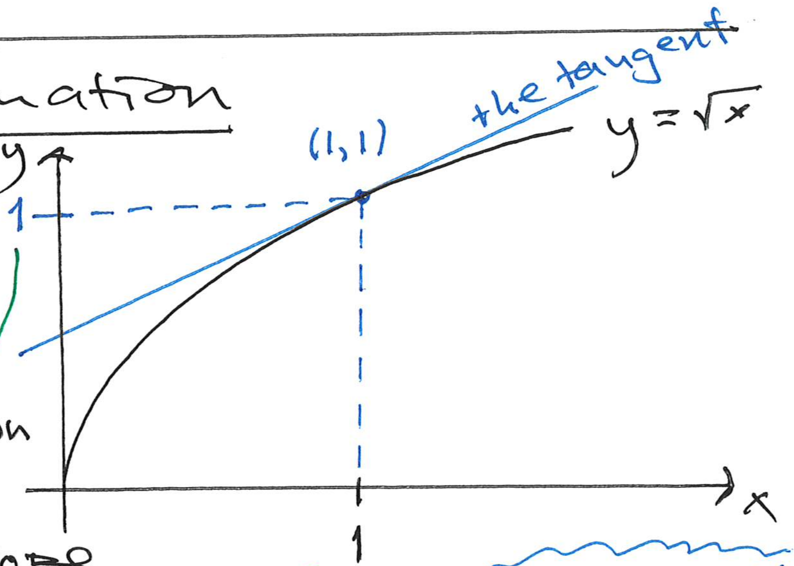
2. Linear approximation

__Ex__ $f(x) = \sqrt{x}$

The linear approximation of $f(x)$ about $x = 1$.

We can find the expression for the tangent line at $x = 1$ by the point-slope formula:



$$y - 1 = f'(1) \cdot (x - 1)$$

so $$y - 1 = \frac{1}{2}(x - 1)$$

or $$y = \underbrace{1}_{= f(1)} + \underbrace{\frac{1}{2}}_{= f'(1)}(x - 1) = P_1(x)$$

$f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2} \cdot x^{\frac{1}{2} - 1}$

$= \frac{1}{2} \cdot x^{-\frac{1}{2}}$

$= \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$

$f'(1) = \frac{1}{2} \cdot \frac{1}{\sqrt{1}}$

$= \frac{1}{2}$

– is called the degree one Taylor polynomial of $\sqrt{x}$ about $x = 1$.

Ex $P_1(1.1) = 1 + \frac{1}{2}(1.1-1) = 1.05$

(check: $\sqrt{1.1} = 1.04881\ldots$)

3. Higher degree Taylor polynomials

Ex $f(x) = \sqrt{x}$

The Taylor polynomial of degree 2 for $\sqrt{x}$ about $x = 1$ is

$$P_2(x) = \underbrace{f(1) + f'(1)\cdot(x-1)}_{P_1(x)} + \frac{f''(1)}{2}(x-1)^2$$

$$= 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2$$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$f''(x) = \frac{1}{2}\cdot\left(-\frac{1}{2}\right)\cdot x^{-\frac{1}{2}-1}$

$= -\frac{1}{4}\cdot x^{-\frac{3}{2}}$

$= -\frac{1}{4x\sqrt{x}}$

$f''(1) = -\frac{1}{4\cdot 1\cdot\sqrt{1}} = -\frac{1}{4}$

Pattern

$$\boxed{P_2(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2}$$

– the second degree Taylor polynomial of a function $f(x)$ about $x = a$.

$$\sqrt{2} = f(2) \approx P_2(2) = 1 + \frac{1}{2}(2-1) - \frac{1}{8}(2-1)^2$$

$$= 1 + \frac{1}{2} - \frac{1}{8} = 1.375$$

(check: $\sqrt{2} = 1.41421\ldots$)

$$P_2(1.2) = 1 + \frac{1}{2}(1.2-1) - \frac{1}{8}(1.2-1)^2$$

$$= 1 + 0.1 - 0.005 = 1.0950$$

(check: $\sqrt{1.2} = 1.0954\ldots$)

Start: 11.05

Ex $f(x)=\sqrt{x}$ about $x=1$. Then the third degree Taylor polynomial for $f(x)$ about $x=1$ is:

$$P_3(x) = \underbrace{P_2(x)}_{\text{already done this!}} + \frac{f'''(1)}{6}\cdot(x-1)^3$$

$$f''(x) = -\frac{1}{4}\cdot x^{-\frac{3}{2}}$$

$$f'''(x) = -\frac{1}{4}\cdot\left(-\frac{3}{2}\right)\cdot x^{-\frac{5}{2}} = \frac{3}{8x^2\sqrt{x}}$$

$$f'''(1) = \frac{3}{8\cdot 1^2\cdot\sqrt{1}} = \frac{3}{8}$$

$$= 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{\left(\frac{3}{8}\right)}{6}\cdot(x-1)^3$$

$$= 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3$$

$$P_3(1.2) = 1 + \frac{1}{2}(1.2-1) - \frac{1}{8}(1.2-1)^2 + \frac{1}{16}(1.2-1)^3$$

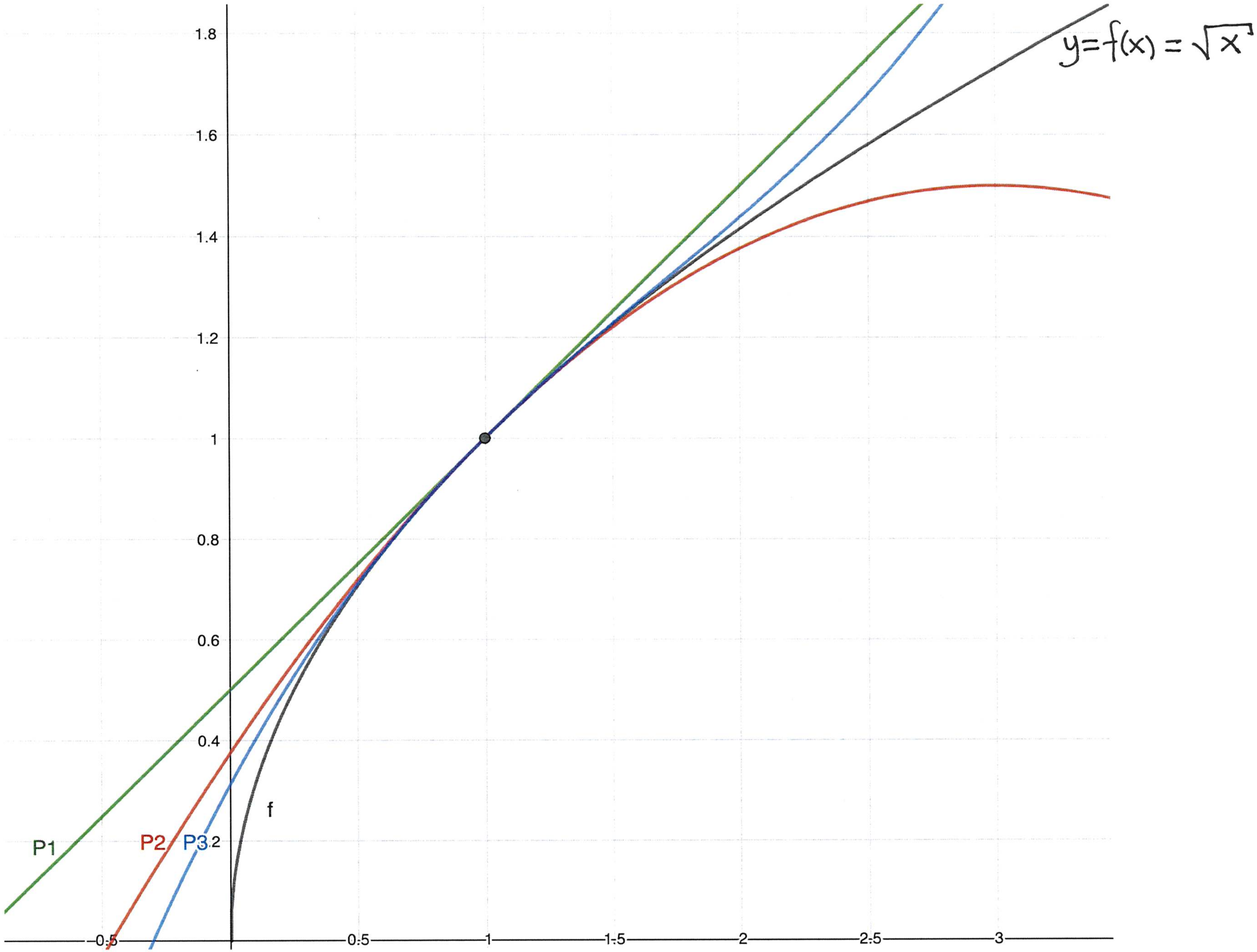
$$= 1.0955$$

Pattern (3rd degree Taylor polynomial about $x=a$)

$$P_3(x) = \underbrace{\overbrace{f(a) + f'(a)(x-a)}^{P_1(x)} + \frac{f''(a)}{2}(x-a)^2}_{P_2(x)} + \frac{f'''(a)}{6}(x-a)^3$$

The degree n Taylor polynomial for $f(x)$ about $x=a$.

$$\boxed{\begin{gathered} P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n \\ \text{where } n! = n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot 3\cdot 2\cdot 1 \end{gathered}}$$


y=f(x) = √x
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
-0.5
0.5
1
1.5
2
2.5
3
f
P1
P2
P3

4. About the exam

- 12 problems of equal weight
  - might have subproblems
- 3 hours (9 - 12), find out where!
- You write your answers (and justifications!) on paper! (3 layers! - cross over, not erase!)

  Advice: One problem per page.
- I am grading.
- The exam counts 40% of the final grade.
- All the problems should be (very) recognizable from tutoring and lectures.
- Many basic and central problems.
- The problems are not ordered according to the lecture plan.
- Support materials: BI-calculator & ruler.
- You have to give justifications!

5. How to prepare

① Relevant material:
- lec. notes
- tutoring problems
- earlier exams
- also some textbook problems

② **My best advice on how to prepare:**

Try to solve the (tutoring-) problems in your head!
- what is the plan? (broad, then in detail!)
- what kind of knowledge is required?
- what kind of obstacles may occur?

③ If I get the wrong answer:
- what went wrong?
  - the plan
  - the calculations

④ When you have solved a problem:
- what did you learn?

⑤ Learn the basics well!
- definitions, concepst ("the words"

⑥ Basic problems are the most important!

Ex $e^x = 5$ or $\ln(x+3) = 2$