EBA 1180
Spring 25
Lect. 1

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→ Info: lecture plan, lecture notes, exercise sheets, messages: It's learning (dr eriksen.no)

Exams: Final exam + retake exams

Topics

→ Integration

→ matrix and vector computations

→ Functions in two variables + everything from last semester

o—o

NEW TOPIC: Integration

Definite integrals

Definition

Def: (Definite integral)

$$\int_a^b f(x)\,dx = \text{area of the region between the graph of } f \text{ and the x-axis in } [a, b]$$

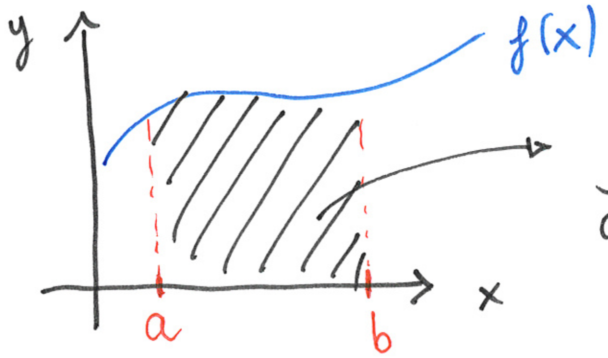
b → "integration bound": upper bound

"integration sign"

$f(x)$: function we're integrating: "the integrand"

a ↑ integration bound: "lower bound"

Read: "Integral from a to b of $f(x)\,dx$"



$$\int_a^b f(x)\,dx$$

ASSUMPTIONS: i) $f(x)$ is a continuous function on $[a,b]$.

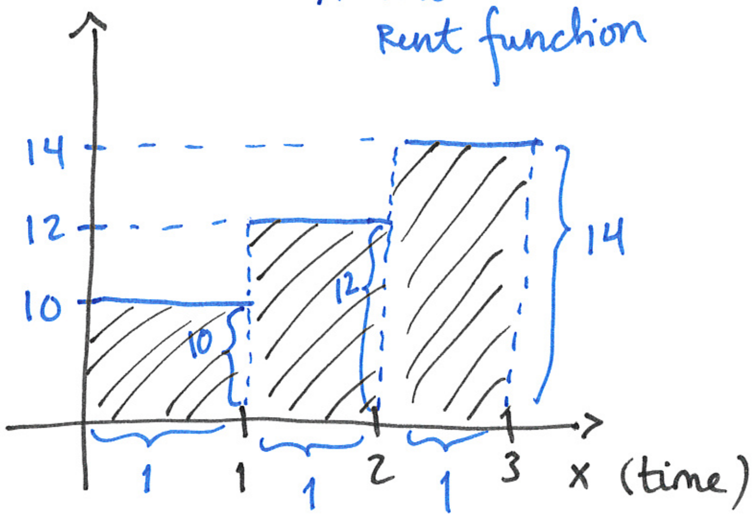
ii) $f(x) \geq 0$ on $[a,b]$

iii) $a < b$

Ex: Rental income over 3 years

Assume: Rent is changed once per year

ASSUME: Rent function



Total rental income = rent year 1 · length year 1 + rent year 2 · length year 2 + rent year 3 · length year 3

$$= 10 \cdot 1 + 12 \cdot 1 + 14 \cdot 1 = 36$$

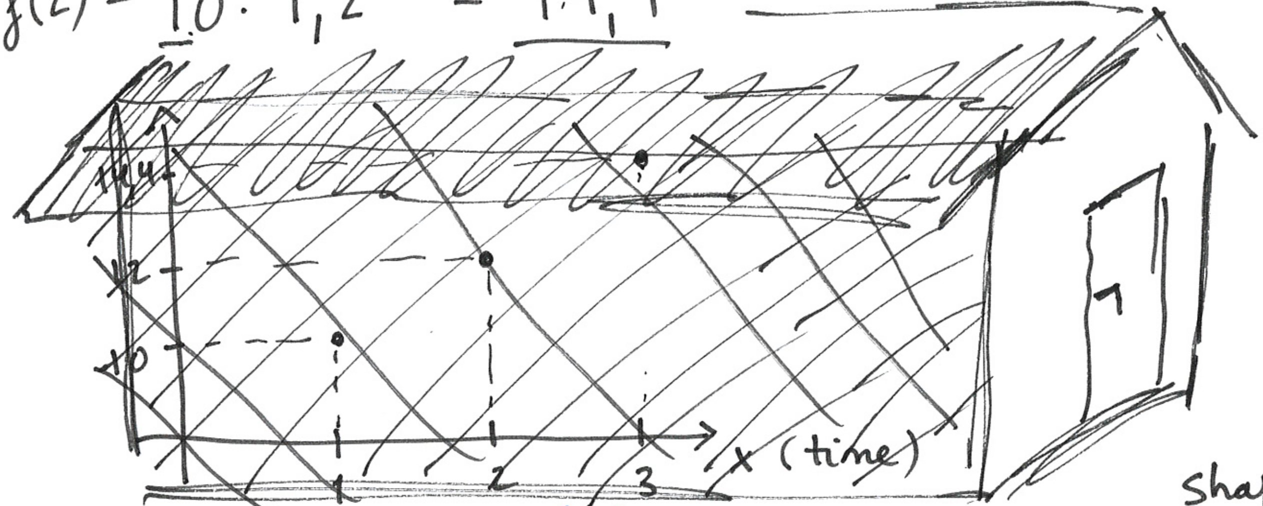
NOW: Assume we have a continuously changing rent:

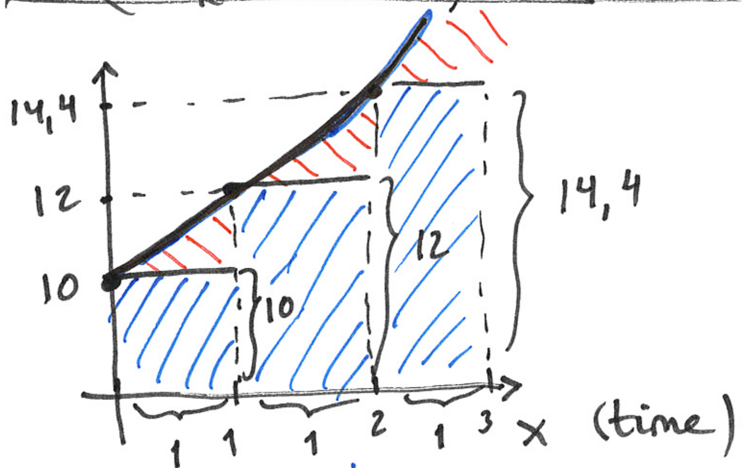
$$\underbrace{f(x)}_{\text{rent}} = 10 \cdot 1,2^{x}$$

$f(0) = 10 \cdot 1,2^{0} = \underline{10}$

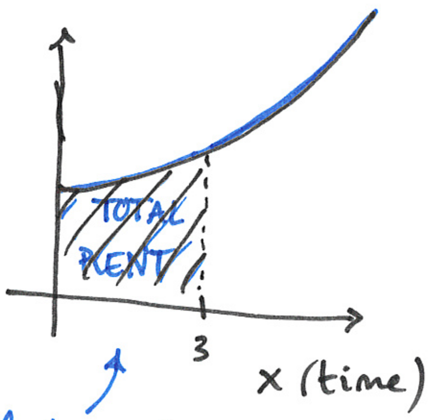
$f(1) = 10 \cdot 1,2^{1} = 10 \cdot 1,2 = \underline{12}$

$f(2) = 10 \cdot 1,2^{2} = \underline{14,4}$





Shape of curve:



Area of rectangles approximates the total rent.

Total rental income over 3 years $\overset{\text{Def.}}{=} \displaystyle\int_0^3 10 \cdot 1,2^{x}\,dx$ = area under graph between 0 and 3

$f(0)\cdot 1 + f(1)\cdot 1 + f(2)\cdot 1$ → Riemann sum

$\approx 10 \cdot 1 + 12 \cdot 1 + 14,4 \cdot 1 = 36,4$

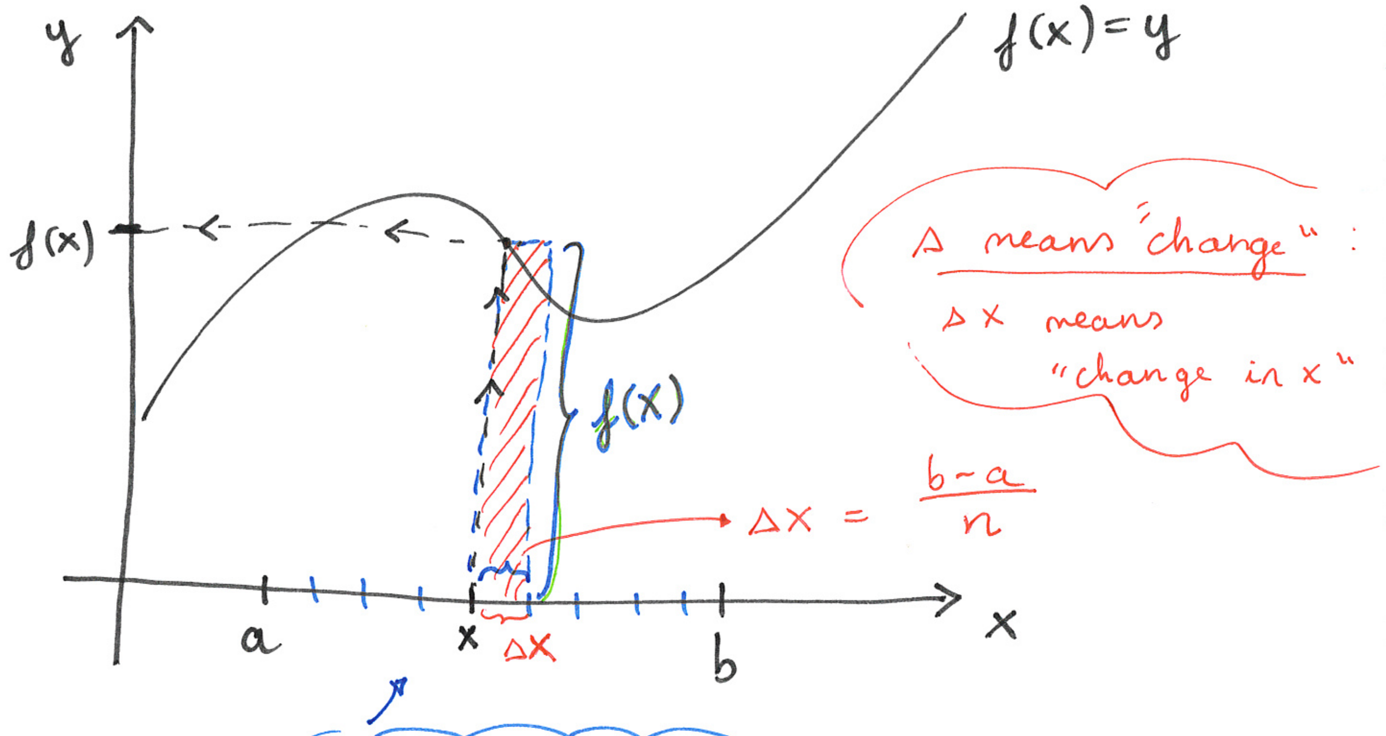
Area of rectangles approximate rental income

NB: Approximation! Missing some bits.

Exercise: How to make approximation better?

Idea: Make width/increments smaller.

NOTE: In general



Rectangle has area:

Pass to limit

$$f(x) \cdot \Delta x \longrightarrow f(x)\underbrace{dx}$$

$n \to \infty$

dx is what we call Δx in the limit

Anti-derivatives and indefinite integrals

Def: (Anti-derivative)
An anti-derivative of a function $f(x)$ is a function $F(x)$ such that $F'(x) = f(x)$.

Ex: $f(x) = 2x \Rightarrow F(x) = x^2$

($f(x) = 4x \xRightarrow{\text{Exercise:}} F(x) = 2x^2$)

because $F'(x) = (x^2)' = 2x = f(x)$

Question: Can you think of another antiderivative of $f(x)$?

Say: $F(x) = x^2 + 1$ because

$$F'(x) = (x^2+1)' = 2x = f(x)$$

More generally: $F(x) = x^2 + C$ because

constant

$$F'(x) = (x^2 + C)' = 2x = f(x)$$

FACT: If $f(x)$ has an antiderivative $F(x)$, then any other antiderivative can be written as $F(x) + C$, where C is a constant.

Def: (Indefinite integral)

The indefinite integral of a function $f(x)$ is

NB: Indefinite since no integration bounds

$$\int f(x)\,dx = \underbrace{F(x) + C}_{\text{All antiderivatives of } f(x)}$$

where $F'(x) = f(x)$.

Ex: $\int 2x\,dx = x^2 + C$

Integration rules

$\boxed{\text{How to compute } \int f(x)\,dx\ ?}$ ⤳ ANTI-DIFFERENTIATE

Ex: $\int 3 + x + x^2\,dx = 3x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + C$

CHECK: Differentiate the answer:

$$\left(3x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + C\right)' = 3 + \frac{1}{\cancel{2}}\cdot\cancel{2}x + \frac{1}{\cancel{3}}\cdot\cancel{3}x^2 = 3 + x + x^2$$

SAME ⇒ OK!

TO PROVE: Differentiate and see you get integrand back: DIY

INTEGRATION RULES

i) Power rule: $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad (n \neq -1)$

CHECK: $\left(\frac{x^{n+1}}{n+1} + C\right)' = \frac{\cancel{(n+1)}\,x^{n+1-1}}{\cancel{n+1}} + 0 = x^n$

ii) $\int \frac{1}{x}\,dx = \ln|x| + C$

iii) Integral of sum is the sum of the integrals:

$$\int u(x) + v(x)\,dx = \int u(x)\,dx + \int v(x)\,dx$$

iv) Multiplied constants can be moved outside the integral:

$$\int c \cdot u(x)\,dx = c \int u(x)\,dx \qquad (c \text{ is a constant})$$

v) Exponentials: $\int e^x dx = e^x + C$ $\quad (e^x)' = e^x$

$$\int a^x dx = \frac{1}{\ln(a)} a^x + C \qquad (a > 0)$$

$(a^x)' = a^x \ln(a)$

NB: Need to add integration constant when indefinite integrals.