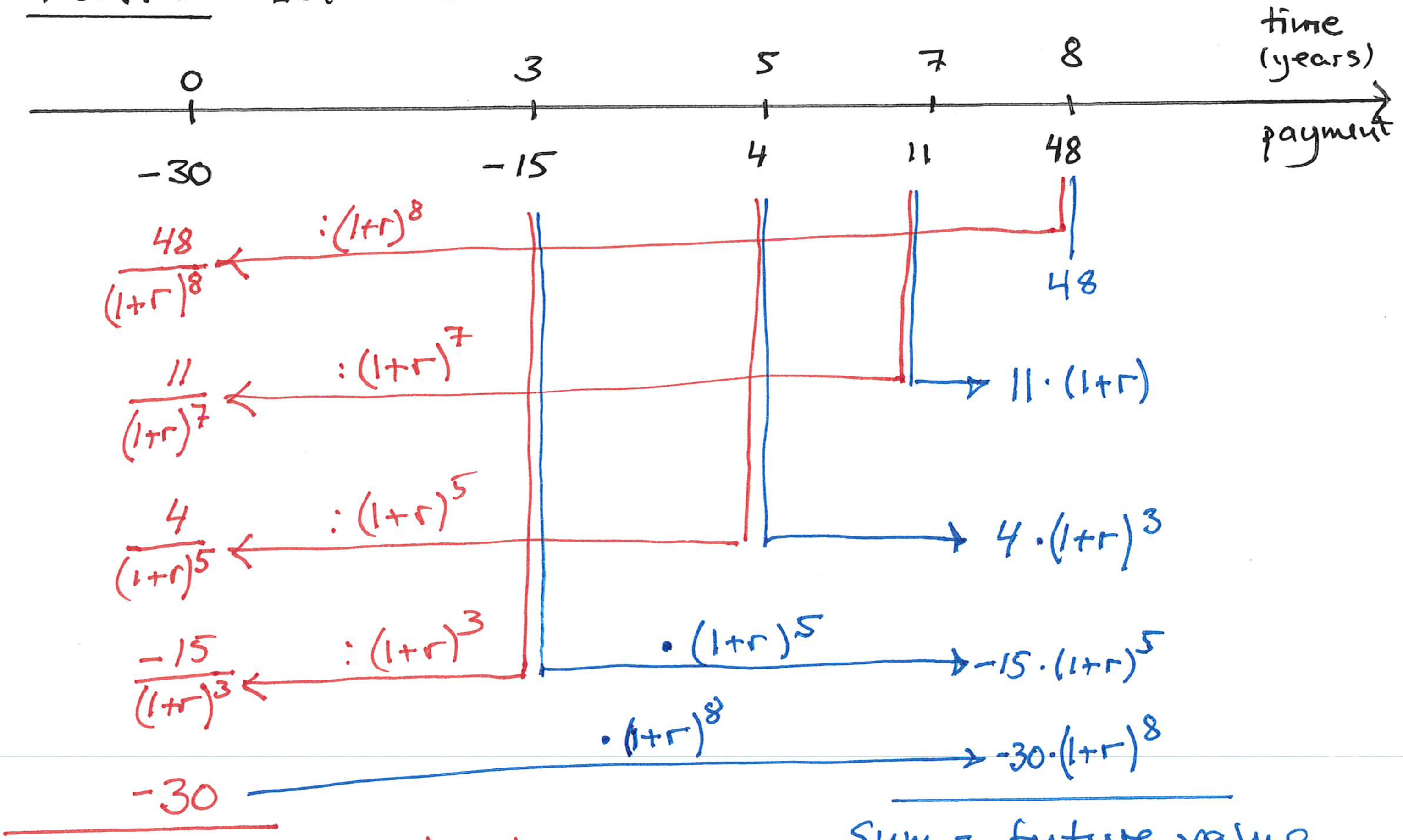
EBA 1180, lecture 4, 3 Sept. 2024, Runar Ile

Plan
1. Repetition: Total present value of a cash flow.
2. Geometric series.
3. Annuities.

## 1. Rep: Total present value.

**Probl. 8** Let $r$ be the interest.




Sum = tot. present value of the cash flow with interest $r$ $= K_0$

Sum = future value of cash flow 8 yrs from now with interest $r$ $= K_8$

b) If $r = 9\%$, the present value $= \underline{\underline{-8.88}}$

a) With $r = 9\%$ the future value $= \underline{\underline{-17.69}}$

d) If $r = 13\%$, the present value $= \underline{\underline{-15.49}}$

c) With $r = 13\%$ the future value $= \underline{\underline{-41.19}}$

Observation $\overset{=K_0}{-8.88} \cdot (1+9\%)^8 \overset{\text{calc.}}{=} -17.69 = K_8$

$$K_0 = -15.49 \cdot (1+13\%)^8 \overset{\text{calc.}}{=} -41.18 \approx K_8$$

e) $K_0 \cdot (1+r)^8 = \left[-30 - \frac{15}{(1+r)^3} + \frac{4}{(1+r)^5} + \frac{11}{(1+r)^7} + \frac{48}{(1+r)^8}\right] \cdot (1+r)^8$

$\uparrow$ tot. pres. value

$$= -30 \cdot (1+r)^8 - 15 \cdot (1+r)^5 + 4 \cdot (1+r)^3 + 11 \cdot (1+r) + 48$$

$= K_8 \leftarrow$ the future value 8 yrs from now with interest r

Problem How much should the payment today (−30) be changed so that IRR of the new cash flow becomes

i) 9 % ? New payment today : $-30 + 8.88 = \underline{\underline{-21.12}}$

ii) 13 % ? —— 〃 —— : $-30 + 15.49 = \underline{\underline{-14.51}}$

How should the payment 8 y's from now (48) be changed so that the future value of the new cash flow 8 yrs. from now becomes 0 if

iii) r=9 % ? New payment 8 yrs from now: $48 + 17.69 = \underline{\underline{65.69}}$

iv) r=13 % ? New payment —〃— : $48 + 41.19 = \underline{\underline{89.19}}$

## 2. Geometric series

A series : - many terms added

third term

Ex $1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{100}$ is a series

with 10 terms.

We write $a_1 + a_2 + a_3 + \ldots + a_{10}$

Start : 11.02

Geometric series: $a_1 + a_2 + \ldots + a_n$
where each term is $k$ times the previous term
($k$ is a fixed number)

$$a_2 = k \cdot a_1$$

$$a_3 = k \cdot a_2 = k \cdot (k \cdot a_1) = k^2 \cdot a_1$$

$$a_4 = k \cdot a_3 = k \cdot (k^2 \cdot a_1) = k^3 \cdot a_1$$

$$\vdots$$

$$a_{10} = k^9 \cdot a_1$$

We can find a short expression for the geom. series (the sum):

$$a_1 + a_2 + \ldots + a_n = a_1 + k \cdot a_1 + k^2 a_1 + \ldots + k^{n-1} \cdot a_n$$

$$= a_1 \underbrace{(1 + k + k^2 + \ldots + k^{n-1})}_{\frac{k^n - 1}{k - 1}}$$

the number of terms

$$= a_1 \cdot \frac{k^n - 1}{k - 1}$$

the first term

'the multiplier' (growth factor)

**Problem** Compute the sum

$$5 + 5\cdot 1.003 + 5\cdot 1.003^2 + \dots + 5\cdot 1.003^{60}$$

**Solution** This is a geometric series with $a_1 = 5$, $k = 1.003$ and the number of terms $n = 61$. Then the sum is

$$5\cdot\frac{1.003^{61}-1}{1.003-1} = 5\cdot\frac{1.003^{61}-1}{0.003} = \underline{\underline{334.14}}$$

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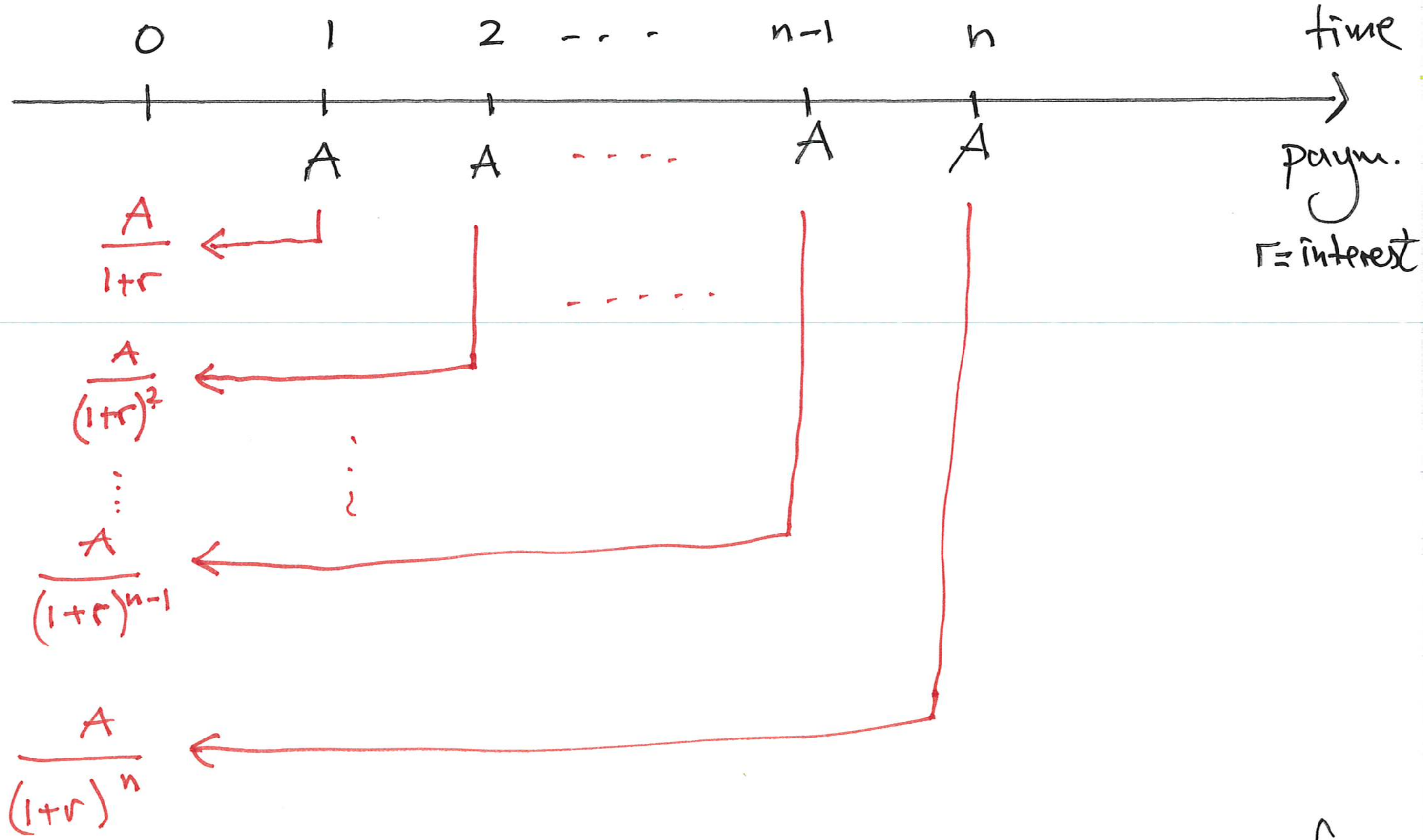
## 3. Annuities

– regular cash flows (same payment, same period length)




The sum = tot. pres. value of the regular cash flow

It is a geometric series with

$$a_1 = \frac{A}{1+r} \quad \text{and} \quad k = \frac{1}{1+r}, \quad n = \text{number of terms}$$

Then the sum is (the tot. pres. value)

$$\frac{A}{1+r} \cdot \frac{\left(\frac{1}{1+r}\right)^n - 1}{\frac{1}{1+r} - 1}$$

not so nice!

BUT: A finite geom. series is also a geom. series in the opposite direction!

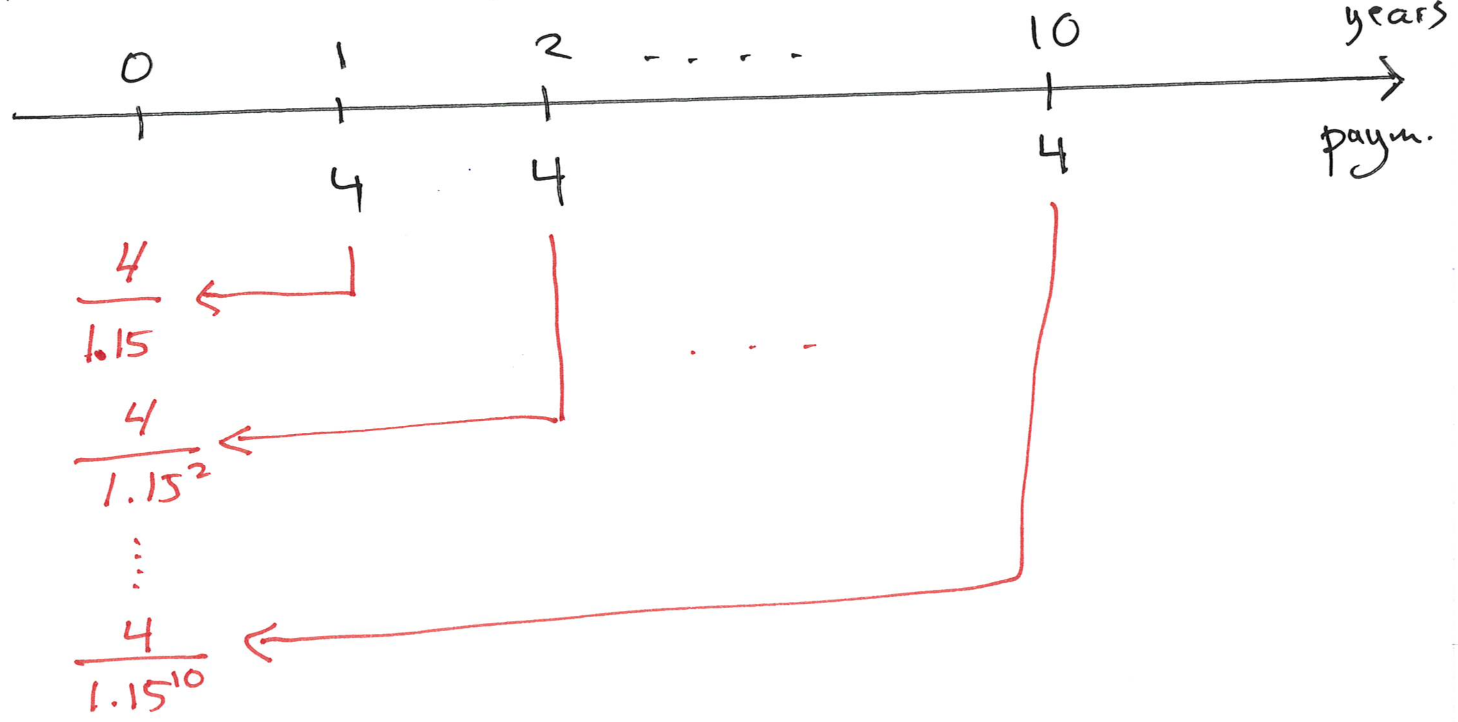
Then $a_1 = \frac{A}{(1+r)^n}$, $k = 1+r$ so the

sum is $$\frac{A}{(1+r)^n} \cdot \frac{(1+r)^n - 1}{1+r-1} = \underbrace{\frac{A}{(1+r)^n} \cdot \frac{(1+r)^n - 1}{r}}$$

– better for calculation

Ex Hege considers an investment where 4 mill is paid out every year for 10 years. The first payment is one year from now.
Suppose the discount rate is 15%
What is a fair price for this investment? (cash flow)

Solution We determine the total pres. value of the cash flow




The sum is a geom. series with

$$a_1 = \frac{4}{1.15^{10}}, \quad k = 1.15, \quad n = 10$$

so the sum (tot. pres. val.) is

$$\frac{4}{1.15^{10}} \cdot \frac{1.15^{10} - 1}{0.15} = \underline{\underline{20.08}}$$