Review of the spring curriculum (a selection)

EBA1180 Spring 25

Part 1: Integration

- Integration methods:

→ Substitution: New variable: $u(x)$

$$du = \underbrace{u'}_{u'(x)}dx$$

$$dx = \frac{1}{u'}du$$

To recognize: Ugly inner function. Perhaps its derivative as a factor.

→ Integration by parts:

$$\int u' \cdot v \, dx = uv - \int u \cdot v' dx$$

Prod. rule: $(u \cdot v)' = u'v + uv'$
(use to derive int. by parts)

To recognize: Product & something nice when anti-diff'ed.

→ Integration of rational functions:

i) $\int \frac{2}{1-x} dx$ ii) $\int \frac{2x}{1-x^2} dx$ iii) $\int \frac{2}{1-x^2} dx$

To solve: i) Substitute $u = 1-x$

ii) Substitute $u = 1-x^2$

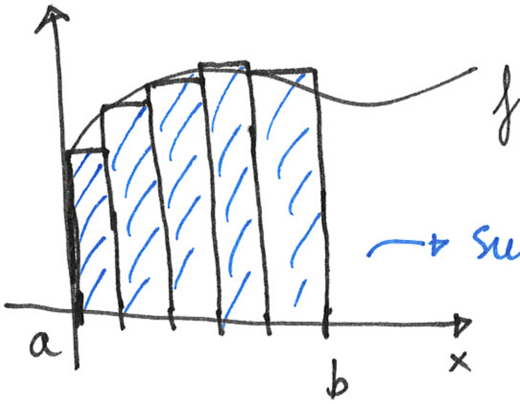
Also works for: $\frac{ax+b}{cx^2+dx+e}$

Factor via abc-formula

iii) Partial fractions: $\frac{2}{\underbrace{1-x^2}_{(1-x)(1+x)}} = \frac{A}{1+x} + \frac{B}{1-x}$; Solve for A and B and type (i) integral

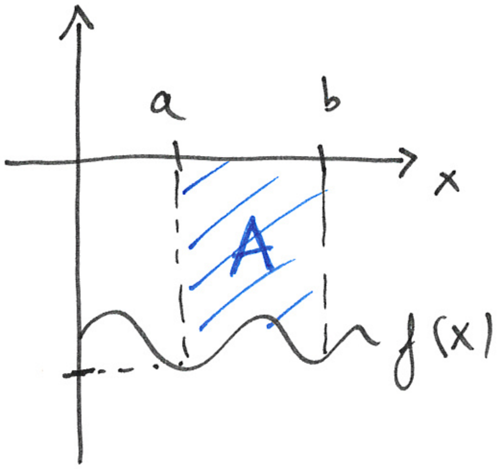
Definite integrals: $$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F'(x) = f(x)$.



sum of area of rectangles $\longrightarrow$ area under graph of f between a and b $= \displaystyle\int_a^b f(x)\,dx$

width of rect. $\to 0$



$$A = - \int_a^b f(x)\,dx$$

OBS!

Economic applications:

Tot. cash flow: $\displaystyle\int_0^T \underbrace{f(x)}_{\text{cash flow per time unit}}\,dx$

NPV of cash flow: $\displaystyle\int_0^T f(x)\, e^{-\overbrace{r}^{\text{discount rate}} x}\,dx$

$\to$ continuous discounting

$$\boxed{\begin{array}{c}\text{What's the plan?}\\ \text{Check your answers!}\end{array}}$$

Part 2: Linear algebra

<u>Thm</u>: Any lin. syst. has either

i) No solutions → Inconsistent

ii) One unique solution
iii) Infinitely many solutions } → Consistent

- $A+B$
- $A-B$
- rA
- $A \cdot B$ ⇒ # columns in A = # rows in B

<u>NB</u>: $A \cdot B \neq B \cdot A$

→ <u>Determinants</u>: For square matrices, $|A| = \det(A)$

<u>2×2</u>: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

<u>n×n</u>: Cofactor expansion along row/column.

<u>NB</u>: Choose wisely! Lots of 0's make life easier.

<u>Result</u>: i) $|A| \neq 0$ ⇒ One unique solution.

ii) $|A| = 0$ ⇒ No solution or infinitely many solutions.

→ Linear combinations: $c_1 \vec{v}_1 + c_2 \vec{v}_2 + c_3 \vec{v}_3$

→ Inverse matrices: Only for square matrices.

A^{-1} s.t. $A^{-1} A = I$

$A A^{-1} = I$

Computing inverses:

$n \times n$: $|A| \neq 0$,

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix}^T$$

where c_{ij} are cofactors.

Alt: $[A \mid I] \sim \ldots \sim [I \mid A^{-1}]$

- $|A| = 0$: No inverse.
- If A^{-1} exists: $A\vec{x} = \vec{b}$

$$A^{-1} A \vec{x} = A^{-1} \vec{b}$$

$$I \vec{x} = A^{-1} \vec{b}$$

$$\vec{x} = A^{-1} \vec{b}$$

Part 3: Functions in two variables

Optimization without constraints

$$\max/\min \; f(x,y)$$

i) Find candidates for max/min:

I) Stationary points: $f'_x = 0$ and $f'_y = 0$

II) Points where f'_x or f'_y not def.

III) Boundary points.

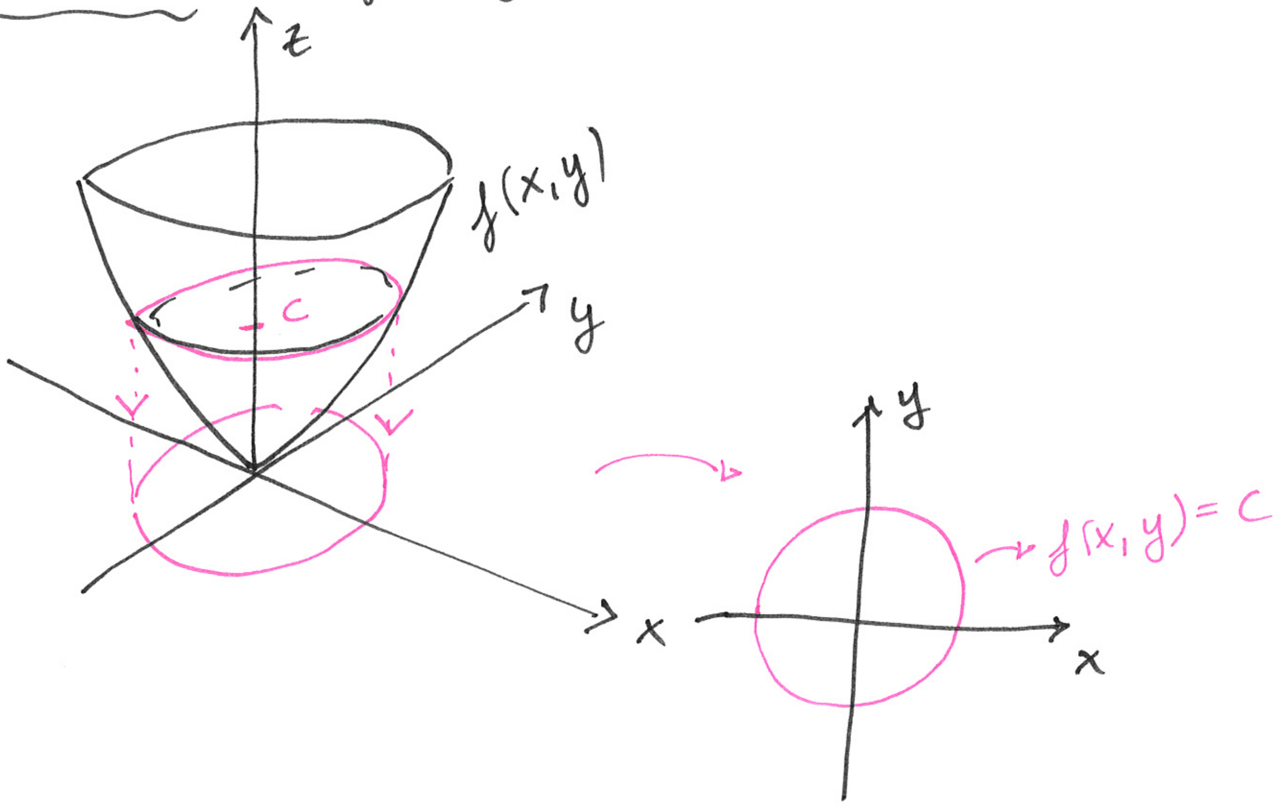
ii) Classify candidates as loc. max/ loc. min/ saddle pts:

2nd derivative test → Not for boundary or pts. where partial deriv. not def.

iii) Analyze whether loc. max./loc. min. are global max/min.

↳ Can  you find smaller/bigger value than local min/max → Not global min/max

Level curves: $f(x, y) = c$



Optimization with constraints

max/min $f(x, y)$ when ...

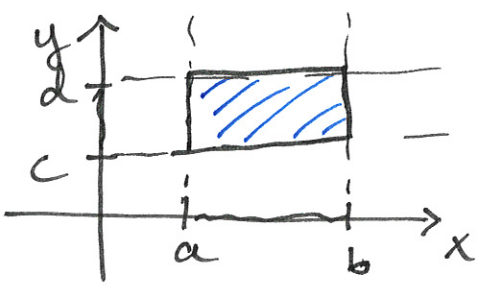
Constraints:
Equations,
inequalities

D = set of admissible points

Extreme value theorem (EVT): f is continuous and D is compact (closed and bounded), then f has a max. and a min. on D.

CASES

I) Optimization on a rectangle: $a \leq x \leq b$
$c \leq y \leq d$



II) Lagrange problems:

$$\max/\min\ f(x,y) \text{ when } g(x,y)=a$$

Candidates for max/min:

i) Lagrange multiplier method:

$$\mathcal{L}(x,y;\lambda) = f(x,y) - \lambda(g(x,y)-a)$$

Lagrange conditions:

$$\text{FOC}\begin{cases} \mathcal{L}'_x = 0 \\ \mathcal{L}'_y = 0 \end{cases} \Rightarrow (x,y;\lambda):$$

$$\text{C}\{\ g(x,y)=a$$

Ordinary candidate points.

ii) Admissible points with degenerate constraints:

$$\boxed{\begin{matrix} g'_x = 0 \\ g'_y = 0 \\ g(x,y)=a \end{matrix}} \Rightarrow (x,y) \text{ candidate}$$

III) Kuhn-Tucker problems: $\max/\min\ f(x,y)$ when $g(x,y) \le a$

Candidates:

i) Boundary: $=$ $\Rightarrow$ Lagrange problem.

ii) Interior stationary pts.

iii) Interior pts. where partial derivatives not def. (e.g. cusps)