EBA1180, lecture 7, 11 Sept. 2024, Runar Ile

Plan
1. Quadratic equations
2. Completing the square
3. Equations with given solutions

1. Quadratic equations

- an eq. which can be transformed into the standard form $ax^2+bx+c=0$ $(a \neq 0)$

It has solution(s):

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Three cases:

$b^2-4ac>0$ gives <u>two</u> solutions

$b^2-4ac=0$ gives <u>one</u> solution

$b^2-4ac<0$ gives <u>no</u> solution

<u>Problem</u> Determine the number of solutions.

a) $x^2+5x+6=0$ $5^2-4\cdot1\cdot6>0$: two solutions

b) $-x^2+2x-1=0$ $2^2-4\cdot(-1)\cdot(-1)=0$: one solution

c) $4x^2-5x-5=0$ $(-5)^2-4\cdot4\cdot(-5)>0$: two solutions

But the abc-formula is often inefficient:

<u>Ex</u> $-3x^2+7=0$ $(a=-3,\ b=0,\ c=7)$

$$-3x^2=-7 \quad |:(-3)$$

$$x^2=\frac{7}{3}$$

$$|x|=\sqrt{x^2}=\sqrt{\frac{7}{3}} \quad \text{so} \quad \underline{\underline{x=\pm\sqrt{\frac{7}{3}}}}$$

Ex $2x^2 - 6x = 0$ $(a = 2,\ b = -6,\ c = 0)$

$2(x^2 - 3x) = 0 \quad | : 2$

$x^2 - 3x = 0$

$x(x-3) = 0$ then

either $\underline{\underline{x = 0}}$ or $x - 3 = 0$

$\underline{\underline{x = 3}}$

Pattern: If $a \cdot b = 0$ then $a = 0$ or $b = 0$ (or both)

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2. Completing the square

Ex $x^2 + 6x - 16 = 0$ that is $x^2 + 6x = 16 \quad | + 9$

Claim: $x^2 + 6x = (x+3)^2 - 9$

- because $(x+3)^2 = x^2 + 2 \cdot 3x + 3^2 = x^2 + 6x + 9$

$6 : 2 =$

so $(x + 3)^2 = 25$

so $x + 3 = 5$ or $x + 3 = -5$

$\underline{\underline{x = 2}}$ $\underline{\underline{x = -8}}$

Problem Solve the quadratic eq. by completing the square

$-4 = \frac{-8}{2}$

a) $x^2 - 8x - 33 = 0$

Solution: Rewrite the eq. $x^2 - 8x = 33 \quad | + (-4)^2 = 16$

and get $\underbrace{(x-4)^2}$ $= 49$ so either $x - 4 = 7$ or $x - 4 = -7$

$x^2 - 8x + 16 = x^2 - 2 \cdot 4x + (-4)^2$

$\underline{\underline{x = 11}}$ $\underline{\underline{x = -3}}$

b) $x^2 + 2x = 63 \quad | +1$

Solution Add $\left(\frac{2}{2}\right)^2 = 1$ to each side

and get $(x+1)^2 = 64$

so $x+1 = 8$ or $x+1 = -8$

$\underline{\underline{x = 7}}$ $\qquad$ $\underline{\underline{x = -9}}$

Start : 11.03

3. Equations with given solutions

Problem Solve the equation $(x-4)\cdot(x+5) = 0$

Solution If a product of two numbers is equal to zero: $a \cdot b = 0$ then at least one of the numbers has to be zero $a = 0$ or $b = 0$

So when $(x-4)\cdot(x+5) = 0$ then

$x-4 = 0$ or $x+5 = 0$

$\underline{\underline{x = 4}}$ $\qquad$ $\underline{\underline{x = -5}}$

Problem Determine the quadratic expression $x^2 + bx + c$ with the given roots (zeros)

a) 1 and 2

Solution: $(x-1)\cdot(x-2) = \underline{\underline{x^2 - 3x + 2}}$

b) 11 and -3

solution: $(x-11)\cdot(x+3) = \underline{\underline{x^2 - 8x - 33}}$

<u>Note</u> $3(x-1)(x-2) = 3x^2 - 9x + 6$ has the same roots as $x^2 - 3x + 2$ (namely $x=1$, $x=2$).

<u>Pattern</u> If r_1 and r_2 are the solutions (roots) the quadratic eq.

$$x^2 + bx + c = 0$$

then $(x - r_1)(x - r_2) = x^2 + bx + c$

so $x^2 - r_2 x - r_1 x + (-r_1)(-r_2) = x^2 + bx + c$

so $x^2 - (r_1 + r_2)x + r_1 r_2 = x^2 + bx + c$

so $-(r_1 + r_2) = b$ and $r_1 r_2 = c$

<u>Ex</u> $x^2 + 6x - 16 = (x-2)(x+8)$ $\quad r_1 = 2$, $r_2 = -8$

<u>Problem</u> Solve the eq.

$$(x^2+1)(12+3x)(9-x^2)(x^2-3x+2) = 0$$

<u>Solution</u> A product equal to zero: one of the factors has to be zero.

$x^2 + 1 = 0$ or $12 + 3x = 0$ or $9 - x^2 = 0$ or $x^2 - 3x + 2 = 0$

$3(4+x) = 0$ $\quad (3-x)(3+x) = 0$ $\quad (x-2)(x-1) = 0$

– no solutions $\quad$ so $\underline{\underline{x = -4}}$ $\quad$ so $\underline{\underline{x = 3}}$, $\underline{\underline{x = -3}}$ $\quad$ so $\underline{\underline{x = 2}}$ or $\underline{\underline{x = 1}}$

Problem Solve the eq. $x^4 - 12x^3 + 11x^2 = 0$

Solution Factorise the LHS: $x^2(x^2 - 12x + 11) = 0$

Either $x^2 = 0$ or $x^2 - 12x + 11 = 0$

$\underline{\underline{x = 0}}$ $\quad$ $(x - 11)(x - 1) = 0$

$\underline{\underline{x = 11}}$ or $\underline{\underline{x = 1}}$