FBA 1180, lecture 8, 17 sept 2024, Runar Ile

Plan 1. Repetition (problems from last week)
2. Polynomial division and factorisation

1. Repetition

2m) Solve the eq. $9x^2 - 6x + 1 = 0$ $\quad 1:9$

$$x^2 - \frac{2}{3}x + \frac{1}{9} = 0$$

Complete the sq: $\left(x - \frac{1}{3}\right)^2 = \left(\frac{1}{3}\right)^2 - \frac{1}{9} = 0$

so $x - \frac{1}{3} = 0$, so $\underline{\underline{x = \frac{1}{3}}}$

Alternative solution: Put $u = 3x$

so $u^2 = (3x)^2 = 3 \cdot x \cdot 3 \cdot x = 3^2 \cdot x^2 = 9x^2$.

So the eq. becomes $u^2 - 2u + 1 = 0$

Complete the sq. $(u-1)^2 = 0$

so $3x = u = 1$

so $\underline{\underline{x = \frac{1}{3}}}$

3e) Determine the quadratic eq. with the solutions

$x = 3 \pm \sqrt{5}$, that is $x = 3+\sqrt{5}$, $x = 3-\sqrt{5}$

Then $(x - (3+\sqrt{5})) \cdot (x - (3-\sqrt{5})) =$

$= x^2 - (3-\sqrt{5})x - (3+\sqrt{5})x + (3+\sqrt{5})(3-\sqrt{5})$

$= x^2 - 6x + 4$ so $\underline{\underline{x^2 - 6x + 4 = 0}}$ has the given solutions.

$(b = -6,\ c = 4)$

5c) Determine k such that the eq.

$\frac{1}{k} \cdot x^2 - 14x = 12$ has exactly one solution

Note: $k \neq 0$, Multiply BS with k:

$$x^2 - 14kx = 12k$$

Complete the sq: $(x - 7k)^2 = 12k + (7k)^2$

has exactly one solution if and only if the RHS = 0, that is

$$12k + 49k^2 = 0$$ - solve for k!

$$k \cdot (12 + 49k) = 0$$

i.e. $k = 0$ or $12 + 49k = 0$

- not allowed!

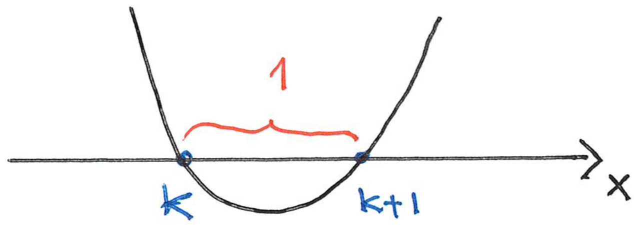
$$\underline{\underline{k = -\frac{12}{49}}}$$

Parametres: numbers without explicit values - used to describe many situation simultaneously.

(economists: "exogeneous variables")

(the x: "endogeneous variable")

Probl. 7a All polynomials x^2+bx+c which have two zeros of distance 1 from each other can be written as



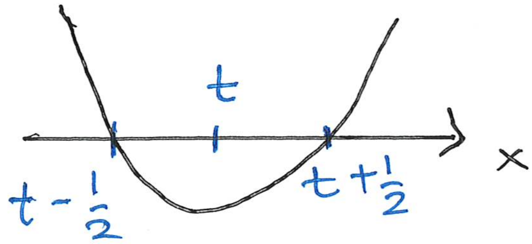
$$\underbrace{(x-k)}^{\text{zero: } x=k}\cdot\underbrace{(x-(k+1))}_{\text{zero: } x=k+1}$$

where k is the smallest zero (root)

Then $(x-k)\cdot(x-(k+1)) = \underline{\underline{x^2-(2k+1)x+k(k+1)}}$

$b=-(2k+1)$, $c=k(k+1)$

Or



and get

$$(x-(t-\tfrac{1}{2}))\cdot(x-(t+\tfrac{1}{2})) = \underline{\underline{x^2-2tx+t^2-\tfrac{1}{4}}}$$

Infinitly many correct solutions (this was just two of them)

Start: 11.01

2. Polynomial division and factorisation

Want to divide a polynomial $f(x)$ with a polynomial $g(x)$ with a remainder $r(x)$.

$$g(x)\cdot \Big| \quad \frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)} \quad \text{with } \deg(r(x)) < \deg(g(x))$$

gives $f(x) = q(x)\cdot g(x) + r(x)$

<u>Ex</u> $f(x) = 3x^2+2x+1$ and $g(x) = x-2$

$$(\boxed{3x^2}+2x+1) : (\boxed{x}-2) = \overset{3x^2:x}{\overset{\shortparallel}{3x}} + \overset{8x:x}{\overset{\shortparallel}{8}} + \frac{17}{x-2}$$

$$\begin{array}{l} -(3x^2-6x \quad) \quad \xleftarrow{\cdot(x-2)} \\ \hline \quad \boxed{8x}+1 \\ \quad -(8x-16) \quad \xleftarrow{\cdot(x-2)} \\ \hline \quad\quad 17 \end{array}$$

17 is called the remainder, $17 = r(x)$

so $\underline{q(x) = 3x+8 \text{ and } r(x) = 17}$

<u>Can check:</u> $\left(3x+8 + \frac{17}{x-2}\right)\cdot(x-2)$

$= (3x+8)(x-2) + \frac{17}{(x-2)}\cdot(x-2)$

$= 3x^2-6x+8x-16+17 = 3x^2+2x+1 = f(x)$

so polynomial division ok!

Two applications of polynomial division

Ⓐ To find asymptotes of rational functions

Ex $\dfrac{3x^2+2x+1}{(x-2)} = 3x+8+\dfrac{17}{x-2}$

has a vertical asymptote: the line $x=2$

and a non-vertical asymptote: the line $y=3x+8$
("oblique")

Ⓑ To factorise a polynomial as a product of degree 1 (linear) polynomials.

Ex Factorise $x^3-4x^2-11x+30$ into linear factors.

Solution Three steps.

step I Guess an integer root (zero).

I try $x=-3$ and get (check):

$(-3)^3-4\cdot(-3)^2-11\cdot(-3)+30$

$=-27-36+33+30 = 0$

Then $(x-(-3)) = (x+3)$ is a factor.

step II Use polynomial division to find a polynomial of lower degree:

$(x^3-4x^2-11x+30):(x+3) \overset{\text{by poly. div.}}{=} x^2-7x+10$

Note: Remainder is 0 !

Step III We find the roots of $x^2-7x+10$

They are $x=2$, $x=5$

so $x^2-7x+10 = (x-2)(x-5)$.

Then $x^3-4x^2-11x+30 = (x-2)(x-5)(x+3)$

Note 1 Not always possible to factorise!

EX x^2+5 has no zeros!

x^2+2x+3 ——— " ——— $(b^2-4ac = 2^2-4\cdot1\cdot3<0)$

Note 2 It can be difficult to guess roots.

– and roots don't have to be integers.