EBA1180 Mathematics for Business Analytics
autumn 2024
Exercises

I came to the position that mathematical analysis is not one of the many ways of doing economic theory: it is the only way.

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Lecture 21 – 22
Sec. 7.12, 6.4, 9.3, 9.5:
l'Hôpital's rule. Marginal revenue and cost.

Here are recommended exercises from the textbook [SHSC].

Section **7.12** exercise 1-3, 4a, 5
Section **6.4** exercise 2, 6
Section **9.5** exercise 1-4

Problems for the exercise session
Wednesday 6 Nov. 12–16+

Problem 1 Compute the limit values.

a) $\lim_{x\to 3} \frac{-x}{25(x-1)}$ b) $\lim_{x\to \ln 5} \frac{e^x-5}{x^2-5}$ c) $\lim_{x\to \ln 5} \frac{e^x-5}{x^2-(\ln 5)^2}$

d) $\lim_{x\to 0} \frac{7x}{e^x-1}$ e) $\lim_{x\to 0} \frac{x^{10}}{e^x-1}$ f) $\lim_{x\to 1} \frac{x\ln(x)}{x^2-1}$

g) $\lim_{x\to 1} \frac{\ln(x)}{e^{2x}-e^2}$ h) $\lim_{x\to 1} \frac{\ln(x)}{\sqrt{x}-1}$ i) $\lim_{x\to 2} \frac{e^{x^2-3x+2}-1}{x^2-4}$

Problem 2 Compute the limit values by applying l'Hôpital's rule.

a) $\lim_{x\to \infty} \frac{-x}{25(x-1)}$ b) $\lim_{x\to 1} \frac{\ln(x)}{2x-2}$ c) $\lim_{x\to \infty} \frac{x^2-4x+10}{e^x-5}$ d) $\lim_{x\to \infty} \frac{\ln(x)}{x}$

Problem 3 Explain why $C(x)$ is a cost function by checking the three criteria:
(1) $C(0) > 0$
(2) $C(x)$ is an increasing function
(3) $C(x)$ is a convex function

Determine the cost optimum and the average cost per unit at cost optimum (also called *the minimal unit cost* or *the optimal unit cost*).

a) $C(x) = 0.01x^2 + 8x + 2500,\ x \geqslant 0$ b) $C(x) = 0.05(x+200)^2,\ x \geqslant 0$

c) $C(x) = 400e^{0.001x^2},\ x \geqslant 0$ d) $C(x) = 50x + 1000,\ 0 \leqslant x \leqslant 1000$

Problem 4 $C(x)$ is the cost function, $R(x)$ is the revenue function and x is number of produced and sold units. Determine the profit maximising number of units.
a) $C(x) = 0.01x^2 + 8x + 2500$ and $R(x) = 100x$ for $x \geqslant 0$
b) $C(x) = 0.005x^2 + 20x + 30000$ and $R(x) = 50x$ for $0 \leqslant x \leqslant 2000$

Problem 5 I figure 1 you see the graph of four different cost functions.

a) Order the cost functions from the one with the smallest minimal unit cost to the one with the largest minimal unit cost.

b) Find an approximate value for the cost optimum for each of the cost functions.

c) Find an approximate value for the minimal unit cost for each of the cost functions.

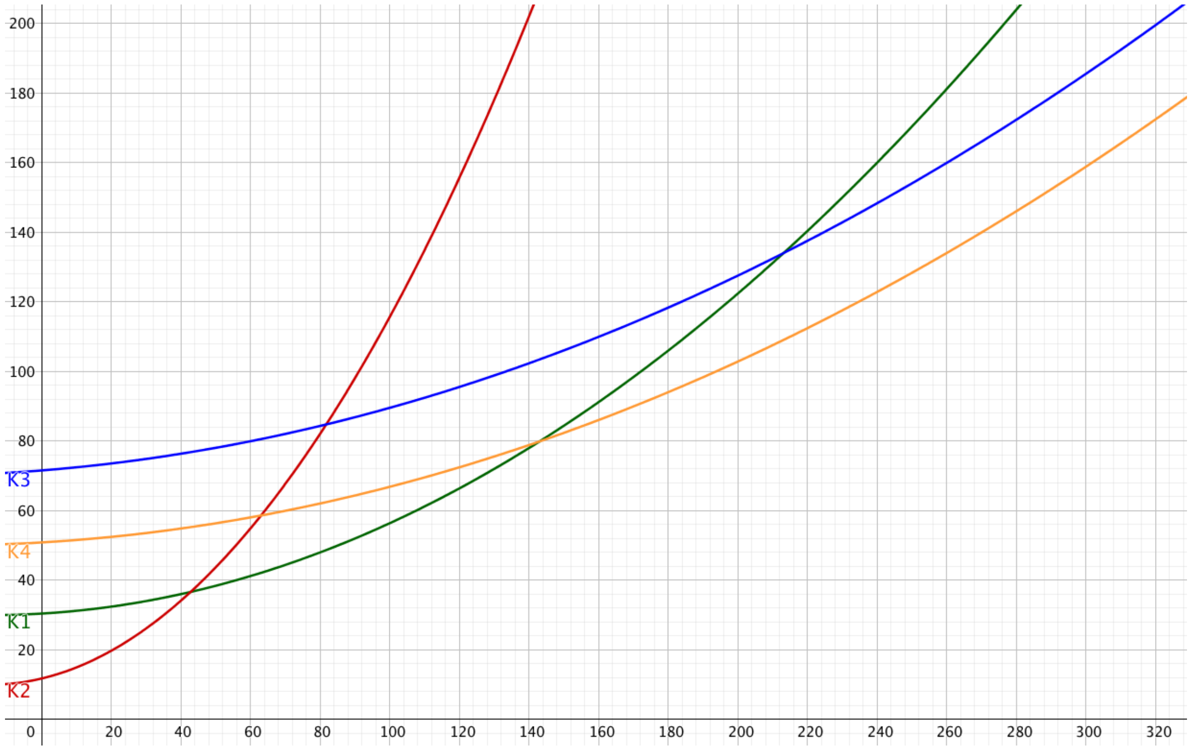

Figure 1: Four cost functions (K_1–K_4)

Problem 6 (Multiple choice exam 2017s, problem 4)
A firm has the cost function $C(x) = 205x^3 - 120x^2 + 2000x + 2800$ when $x \geqslant 0$. What is the minimal average unit cost?

(A) 2 kr
(B) 12 kr
(C) 3980 kr
(D) 7960 kr
(E) I choose not to answer this problem.

Problem 7 (Multiple choice exam 2016a, problem 14)
We consider the limit value

$$\lim_{x\to\infty} \frac{1 - x\ln(x)}{e^x}$$

What is true?

(A) The limit value does not exist
(B) The limit value equals 1
(C) The limit value equals $-\frac{1}{2}$
(D) The limit value equals 0
(E) I choose not to answer this problem.

Problem 8 (Multiple choice exam 2015a, problem 15)
We consider the limit value

$$\lim_{x \to 1} \frac{\ln(x) - x + 1}{x^2 - 2x + 1}$$

What is true?

(A) The limit value does not exist
(B) The limit value equals 0
(C) The limit value equals 1
(D) The limit value equals $-\frac{1}{2}$
(E) I choose not to answer this problem.

Problem 9 (Multiple choice exam 2018a, problem 14)
We have a curve implicitly defined by the equation $4x^2 - 7xy + 4y^2 = 16$.
Which statement is correct?

(A) There is only one point on the curve with x-coordinate 4 and the slope of the tangent at this point is equal to -1
(B) There are two points on the curve with x-coordinate 4 and the product of the slopes of the tangents at these points is -2.75
(C) There are two points on the curve with x-coordinate 4 and the product of the slopes of the tangents at these points is -64
(D) There are two points on the curve with x-coordinate 4 and the product of the slopes of the tangents at these points is $\frac{1024}{425}$
(E) I choose not to answer this problem.

Answers

Problem 1

a) $\frac{-3}{25(3-1)} = -0.06$ b) 0 c) $\frac{5}{2\ln 5}$

d) 7 e) 0 f) 0.5

g) $\frac{1}{2e^2}$ h) 2 i) $\frac{1}{4}$

Problem 2

a) $\frac{-1}{25}$ b) $\frac{1}{2}$ c) $\lim_{x\to\infty} \frac{2}{e^x} = 0$ d) 0

Problem 3

a) $C(0) = 2500 > 0$, $C'(x) = 0.02x + 8 > 0$ for $x > 0$ and so $C(x)$ is an increasing function for $x \geqslant 0$, $C''(x) = 0.02 > 0$ and so $C(x)$ is a convex function for $x \geqslant 0$. Cost optimum $x = 500$ gives minimal unit cost $A(500) = 18$

b) $C(0) = 2000 > 0$, $C'(x) = 0.1x + 20 > 0$ for $x > 0$ and so $C(x)$ is a increasing function for $x \geqslant 0$, $C''(x) = 0.1 > 0$ and so $C(x)$ is a convex function for $x \geqslant 0$. Cost optimum $x = 200$ gives minimal unit cost $A(200) = 40$

c) $C(0) = 400 > 0$, $C'(x) = 0.8xe^{0.001x^2} > 0$ for $x > 0$ and so $C(x)$ is an increasing function for $x \geqslant 0$, $C''(x) = 0.8(1 + 0.002x^2)e^{0.001x^2} > 0$ and so $C(x)$ is a convex function for $x \geqslant 0$. Cost optimum $x = 22.36$ gives minimal unit cost $A(22.36) = 29.49$

d) $C(0) = 1000 > 0$, $C'(x) = 50 > 0$ and so $C(x)$ is an increasing function for $x \geqslant 0$, $C''(x) = 0 \geqslant 0$ and so $C(x)$ is a convex function for $x \geqslant 0$. Cost optimum $x = 1000$ gives minimal unit cost $A(1000) = 51$

Problem 4

a) For $x = 4600$ the marginal cost equals the marginal revenue and $\pi''(x) = -0.02 < 0$ gives that the profit function is concave and hence $x = 4600$ is maximising the profit.

b) For $x = 3000$ the marginal cost equals the marginal revenue, but this is outside the domain of definition for the modell. We see that $\pi'(x) = 30 - 0.01x$ is positive for $x < 3000$ which gives that the profit function is increasing for x in the interval $[0, 2000]$ and hence $x = 2000$ is maximising the profit.

Problem 5

a) K_4, K_1, K_3, K_2

b) $K_4:\ x = 220,\ K_1:\ x = 120,\ K_3:\ x = 270,\ K_2:\ x = 40$

c) $A_4(220) = \frac{112}{220} = 0.51,\ A_1(120) = \frac{65}{120} = 0.54,\ A_3(270) = \frac{165}{270} = 0.61,\ A_2(40) = \frac{35}{40} = 0.88$

Problem 6 (Multiple choice exam 2017s, problem 4)
C

Problem 7 (Multiple choice exam 2016a, problem 14)
D

Problem 8 (Multiple choice exam 2015a, problem 15)
D

Problem 9 (Multiple choice exam 2018a, problem 14)
B