## Exercise session problems

**Problem 1.**
Consider the function $f(x,y) = \ln(2x - y)$ close to the point $(x,y) = (1,1)$.

a) Compute $c = f(1,1)$, and draw the level curves $f(x,y) = c$ and $f(x,y) = c+1$ in the same coordinate system.
b) Compute $\nabla f(1,1)$, and draw this vector into the figure with starting point $(1,1)$.
c) Determine the linear approximation of $f(x,y)$ close to $(1,1)$, and use this to estimate $f(1.1,0.9)$.
d) Find the equation for the tangent plane of the graph $z = f(x,y)$ in $(1,1)$.

**Problem 2.**
Determine the (global) maximum- and minimum values of $f(x,y) = \sqrt{1 - 4x^2 - 9y^2}$, if they exist.

**Problem 3.**
Use the Lagrange multiplier method to find candidates for the maximum and/or minimum:

a) $\max / \min f(x,y) = 3x - y$ when $x^2 + 4y^2 = 37$
b) $\max / \min f(x,y) = x^2 + 4y^2$ when $3x - y = 37$
c) $\max / \min f(x,y) = xy$ when $x^2 + 4y^2 = 8$
d) $\max / \min f(x,y) = 4x^2 + 9y^2$ when $xy = 6$
e) $\max f(x,y) = x^2y^2 - x^2 - y^2 + 16$ when $x^2 + y^2 = 16$
f) $\max f(x,y) = x^2y^2 - x^2 - y^2 + 16$ when $xy = 4$

**Problem 4.**
Find the maximum/minimum, if it exists:

a) $\max / \min f(x,y) = 3x - y$ when $x^2 + 4y^2 = 37$
b) $\max / \min f(x,y) = x^2 + 4y^2$ when $3x - y = 37$
c) $\max / \min f(x,y) = xy$ when $x^2 + 4y^2 = 8$
d) $\max / \min f(x,y) = 4x^2 + 9y^2$ when $xy = 6$
e) $\max f(x,y) = x^2y^2 - x^2 - y^2 + 16$ when $x^2 + y^2 = 16$
f) $\max f(x,y) = x^2y^2 - x^2 - y^2 + 16$ when $xy = 4$

**Problem 5.**
Solve the Lagrange problem: $\max U(x,y) = 0.3\ln(x - 3) + 0.7\ln(y - 2)$ when $12x + 5y = 60$.

**Problem 6.**
**Exam MET1180 (December 2015) Exercise 5**
Consider the level curve $g(x,y) = 0$, where $g$ is the function $g(x,y) = x^3 + xy + y^2$.

a) Find all points on the level curve with $x = -2$, and determine the tangent in each of these points.
b) Find the maximum value of $f(x,y) = x$ under the constraint $x^3 + xy + y^2 = 0$.

**Problem 7.**
**Exam MET1180 (June 2016) Exercise 5**
Consider the Lagrange problem $\max / \min f(x,y) = x + 2y - \sqrt{36 - x^2 - 4y^2}$ when $x^2 + 4y^2 = 36$.

a) Find the points on the level curve $x^2 + 4y^2 = 36$ where the tangent has slope $y' = 1/2$.
b) Make a sketch of $D = \{(x,y) : x^2 + 4y^2 = 36\}$. Is $D$ bounded? What kind of curve is this?
c) Solve the Lagrange problem and find the maximum- and minimum value.
d) Solve the new optimization problem we get when we change the constraint to $x^2 + 4y^2 \leq 36$.

**Problem 8.**
**Difficult!**
Solve the Lagrange problem $\max f(x,y) = x + y$ when $x^3 - 3xy + y^3 = 0$. You can assume that the problem has a maximum.

## Optional: Exercises from the Norwegian textbook

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|---|---|
| Textbook [E]: | Eriksen, *Matematikk for økonomi og finans* |
| Exercise book [O]: | Eriksen, *Matematikk for økonomi og finans - Oppgaver og Løsningsforslag* |

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|---|---|
| Exercises: | [E] 7.6.3 - 7.6.6 |
| Solution manual: | See [O] Ch. 7.6 |

## Answers to the exercise session problems

**Problem 1.**

a) $c = 0$, the level curves are straight lines.
b) $\nabla f(1,1) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$
c) $f(x,y) \approx 2(x-1) - (y-1)$, $f(1.1,0.9) \approx 0.3$
d) $z = 2x - y - 1$

**Problem 2.**
$f_{\max} = 1$, $f_{\min} = 0$

**Problem 3.**
a) $(x,y;\lambda) = (6,-1/2;1/4)$, $(-6,1/2;-1/4)$
b) $(x,y;\lambda) = (12,-1;8)$
c) $(x,y;\lambda) = (2,1;1/4)$, $(-2,-1;1/4)$, $(2,-1;-1/4)$, $(-2,1;-1/4)$
d) $(x,y;\lambda) = (3,2;12)$, $(-3,-2;12)$
e) $(x,y;\lambda) = (\pm 2\sqrt{2}, \pm 2\sqrt{2}; 7), (\pm 4, 0; -1), (0, \pm 4; -1)$
f) $(x,y;\lambda) = (2,2;6), (-2,-2;6)$

**Problem 4.**

a) $f_{\max} = 37/2,\ f_{\min} = -37/2$

b) $f_{\min} = 148$ (does not have a maximum)

c) $f_{\max} = 2,\ f_{\min} = -2$

d) $f_{\min} = 72$ (does not have a maximum)

e) $f_{\max} = 64,\ f_{\min} = 0$

f) $f_{\max} = 24$ (does not have a minimum)

**Problem 5.**

We find the maximum point $(x,y) = (67/20, 99/25)$, maximum value $f_{\max} = 1.7\ln(1.4) - 0.6\ln(2)$ with $\lambda = 1/14$.

**Problem 6.**

a) $y = -8x/3 - 4/3$ i $(-2,4)$ and $y = 5x/3 + 4/3$ i $(-2,-2)$

b) $f_{\max} = 1/4$

**Problem 7.**

a) $(3\sqrt{2}, -3\sqrt{2}/2), (-3\sqrt{2}, 3\sqrt{2}/2)$

b) Yes, ellipse with half axes $a = 6$ and $b = 3$ with center $(0,0)$

c) $f_{\max} = 6\sqrt{2},\ f_{\min} = -6\sqrt{2}$

d) $f_{\max} = 6\sqrt{2},\ f_{\min} = -6\sqrt{3}$

**Problem 8.**

$f_{\max} = 3$