Exercise session problems

Problem 1.
Consider the Lagrange problem

$$\min f(x,y) = x^2 + y^2 - 4y \quad \text{when} \quad 2x + y^2 = -1$$

a) Sketch the curve $2x + y^2 = -1$, and determine whether this is a bounded set.
b) Write down the Lagrange conditions, and find all points $(x,y;\lambda)$ which satisfy these conditions.
c) Solve the Lagrange problem and find the minimum value, if it exists.

Problem 2.
Consider the function defined by $f(x,y) = y^2 - x^3 + 3x$, and denote the level curve of f which passes through the point $(x,y) = (-1,2)$ by C.

a) Find all stationary points of f, and classify these points.
b) Find the tangent of C in the point $(x,y) = (-1,2)$. Does the tangent intersect C in any other points?
c) Sketch the curve in the xy-plane given by $4x^2 + y^2 = 4$. What kind of curve is this? Is it bounded?
d) Solve the optimization problem: $\max f(x,y) = y^2 - x^3 + 3x$ when $4x^2 + y^2 = 4$

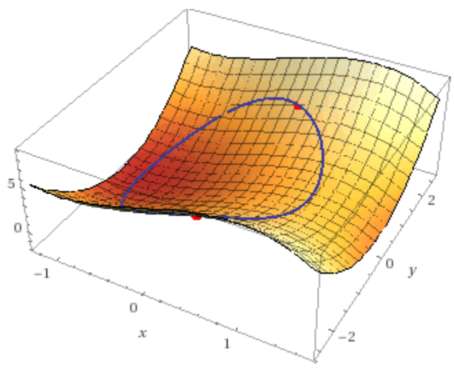


Figure 1: Illustration for Problem 2

Problem 3.
Solve the Lagrange problem: $\min f(x,y) = x$ når $y^2 - x^3 + 3x = 2$

Problem 4.
Consider the function defined by $f(x,y) = x^2 + y^2 - x^2y^2$.

a) Find all stationary points for f, and classify them.
b) Find the global maximum- and minimum values for f, if they exist.
c) Solve the optimization problem: $\min f(x,y) = x^2 + y^2 - x^2y^2$ when $xy = 1$.
d) Estimate the minimum value of $\min f(x,y) = x^2 + y^2 - x^2y^2$ when $xy = a$.

Problem 5.

Consider the function defined by $f(x,y) = x^2y + xy^3 + xy^2$.

a) Compute f'_x and f'_y and find the stationary points of f.

b) Is (0,0) a saddle point? Give reasons for your answer.

c) Find all local maxima and minima for f.

d) Let $D = \{(x,y) : 0 \le x \le 1 \text{ and } 0 \le y \le 1\}$. Find the maximum- and minimum value of f over D.

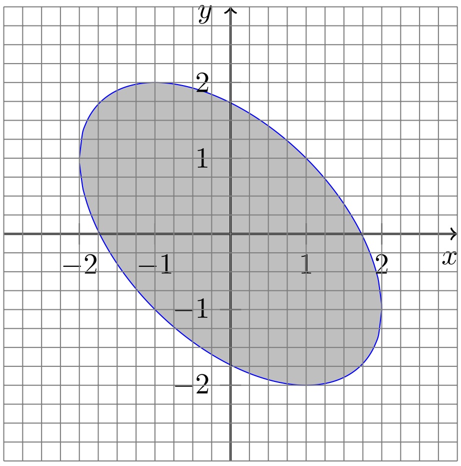
Problem 6.

In the figure below, the blue curve is given by the equation $g(x,y) = a$, and the grey area is given by the inequality $g(x,y) \le a$. Consider the maximization problem

$$\max f(x,y) = x + y \text{ when } g(x,y) \le a$$

a) Show that the maximization problem has a solution which is on the blue curve.

b) Use the figure to estimate the maximum value. Give reasons for your answer.



Answers to the exercise session problems

Problem 1.

a) Parabola (rotated). Not bounded.

b) $(-1{,}1; -1)$.

c) $f_{\min} = -2$.

Problem 2.

a) (1,0) saddle point, $(-1{,}0)$ local minimum

b) $y = 2$, also intersects in (2,2).

c) Ellipsis, bounded.

d) $f_{\max} = 122/27$.

Problem 3.
$f_{\min} = -2$.

Problem 4.

a) (0,0) local minimum, $(\pm 1, \pm 1)$ four saddle points.

b) Neither maximum nor minimum.

c) $f_{\min} = 1$.

d) $f^*(a) \approx 1$ for a close to 1.

Problem 5.

a) (0,0), $(0, -1)$, $(3/25, -3/5)$.

b) Yes.

c) $(3/25, -3/5)$ local maximum.

d) $f_{\max} = 3$, $f_{\min} = 0$.

Problem 6.

a) Compact, no stationary points.

b) $f_{\max} \approx 2$.