

EBA1180 Mathematics for Data Science
autumn 2024
Exercises

... if I couldn't formulate a problem in economic theory mathematically, I didn't know what I was doing.

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Lecture 6 and 7

Sec. 3.1-3, 4.7:

Linear and quadratic equations. Equations with parametres. Polynomial equations.

Here are recommended exercises from the textbook [SHSC].

Section 3.1 exercise 1-5

Section 3.2 exercise 2a, b, e, 4a, 5a, c

Section 3.3 exercise 1, 2, 6

Problems for the exercise session
Wednesday 11 Sept. at 12-16+ in CU1-067

Problem 1 Solve the linear equations for x .

a) $3x - 5 = 0$

b) $ax + b = 0$

c) $cx + 50 - c = 0$ ($c \neq 0$)

d) $-4x + 9 = 3 - 0,25x$

e) $ax + b = cx + d$

f) $\frac{x}{4} + 11 - k = 3 - \frac{x}{3}$

g) $22 - 4x + 9 - 2(x + 7) = 14 - 5(x + 2) + x - 23$

h) $k - bx - d(3 - 5x) + 11 = c$

i) $\frac{4}{x-2} = \frac{3}{2x+1}$

j) $\frac{5-x}{x-a} = 7$

k) $\frac{6}{3x+6} = \frac{14}{2x+4}$

l) $5 - 7x = 4x + 29 - 11x$

m) $3(x + 1) = 3x + 3$

Problem 2 Solve the quadratic equations (without using the abc -formula).

a) $x^2 = 0$

b) $x^2 = 1$

c) $x^2 = -1$

d) $(x - 3)^2 = 0$

e) $(x + 3)^2 = 1$

f) $(x + 3)^2 = -1$

g) $x^2 - 6x + 9 = 0$

h) $x^2 - 6x + 9 = 1$

i) $x^2 - 6x + 9 = -1$

j) $(3x - 1)^2 = 0$

k) $(3x - 1)^2 = 9$

l) $(3x - 1)^2 = -9$

m) $9x^2 - 6x + 1 = 0$

n) $9x^2 - 6x + 1 = 9$

o) $9x^2 - 6x + 1 = -9$

p) $(x - 2)(x + 3) = 0$

q) $(3x + 5)(2x - 1) = 0$

r) $(x - 2)(x + 3) = -6$

s) $x^2 - 5x - 2 = 0$

t) $2x^2 + 4x = 70$

u) $x(x - 8) = 90 - x^2$

Problem 3 Determine the quadratic equation $x^2 + bx + c = 0$ with the given solutions.

a) $x = 1, x = -1$

b) $x = 3, x = -2$

c) $x = -3, x = -8$

d) $x = 17, x = -3$

e) $x = 3 \pm \sqrt{5}$

f) $x = -11$

Problem 4 Solve the quadratic equations by *completing the square*.

a) $x^2 - 8x = -12$

b) $x^2 + 10x = -9$

c) $x^2 - 5x - 14 = 0$

d) $x^2 - 3x = 4$

e) $x^2 - 24x = 25$

f) $x^2 + 5x = 6$

Problem 5

a) Determine k such that the equation $x^2 - 10x = k$ has exactly one solution.

b) Determine k such that the equation $x^2 - kx = -9$ has exactly one solution.

c) Determine k such that the equation $\frac{1}{k}x^2 - 14x = 12$ has exactly one solution.

Problem 6 Write the quadratic expression as a product of two linear (degree 1) polynomials.

a) $x^2 - 8x + 12$

b) $x^2 + 10x + 9$

c) $x^2 - 18x + 81$

d) $2x^2 - 4x - 70$

e) $3x^2 + 24x - 99$

f) $5x^2 + 60x + 180$

Problem 7

a) Use one parameter to write an expression for all polynomials on the form $x^2 + bx + c$ which have two zeros of distance 1 from each other.

b) Use one parameter to write an expression for all third degree polynomials on the form $x^3 + ax^2 + bx + c$ which have three zeros with the middle one of distance 5 to each of the other two.

c) Use two parameters to write up all quadratic polynomials with exactly one zero.

Problem 8 Solve the equations for x .

a) $(2x - \sqrt{3})(x^2 - 20x + 99) = 0$

b) $(x^2 - 5)(x^2 - 6x + 4) = 0$

c) $(2x^2 - 28x + 98)(x^2 - 6x + 10)(x^2 - 2x - 6) = 0$

d) $(x + 1)(x + 2)(x + 3)(x + a)(x^2 + b) = 0$

Problem 9 Solve the equations.

a) $x^4 - 11x^2 + 18 = 0$

b) $x^6 + 13x^3 + 40 = 0$

c) $x^{100} - x^{50} - 12 = 0$

Problem 10 Solve the equations.

a) $1 + x + x^2 + x^3 + \dots + x^{98} + x^{99} = 0$

b) $1 + x + x^2 + x^3 + \dots + x^{49} + x^{50} = 0$

c) $\frac{x}{1.02} + \frac{x^2}{1.02^2} + \frac{x^3}{1.02^3} + \dots + \frac{x^{29}}{1.02^{29}} + \frac{x^{30}}{1.02^{30}} = 0$

Answers

Problem 1

- a) $x = \frac{5}{3}$ b) $x = -\frac{b}{a}$ c) $x = \frac{c-50}{c} = 1 - \frac{50}{c}$ d) $x = \frac{8}{5}$
- e) $x = \frac{d-b}{a-c}$ if $a \neq c$. For $a = c$ and $b \neq d$ there is no solution,
for $a = c$ and $b = d$ all numbers are solutions.
- f) $x = \frac{12k-96}{7}$ g) $x = 18$
- h) $x = \frac{c+3d-k-11}{5d-b}$ if $b \neq 5d$. If $b = 5d$ and $c + 3d \neq k + 11$ there are no solutions,
if $b = 5d$ and $c + 3d = k + 11$ all numbers are solutions.
- i) $x = -2$ j) $x = \frac{7a+5}{8}$ k) no solutions l) no solutions
- m) all numbers are solutions

Problem 2

- a) $x = 0$ b) $x = 1, x = -1$ c) no solutions d) $x = 3$
- e) either $x + 3 = 1$, i.e. $x = -2$,
or $x + 3 = -1$, i.e. $x = -4$ f) no solutions g) $x = 3$
- h) $x = 2, x = 4$ i) no solutions j) $x = \frac{1}{3}$ k) $x = \frac{1}{3} \pm 1$, i.e.
 $x = \frac{4}{3}, x = -\frac{2}{3}$
- l) no solutions m) $x = \frac{1}{3}$ n) $x = \frac{4}{3}, x = -\frac{2}{3}$ o) no solutions
- p) $x = 2, x = -3$ q) $x = -\frac{5}{3}, x = \frac{1}{2}$ r) $x = 0, x = -1$ s) $x = \frac{5}{2} \pm \frac{\sqrt{33}}{2}$
- t) $x = -7, x = 5$ u) $x = -5, x = 9$

Problem 3

- a) $(x-1)(x-(-1)) = x^2 - 1$ b) $x^2 - x - 6$ c) $x^2 + 11x + 24$
- d) $x^2 - 14x - 51$ e) $x^2 - 6x + 4$ f) $x^2 + 22x + 121$

Problem 4

- a) $(x-4)^2 = -12 + 16$ gives
 $x = 2, x = 6$ b) $(x+5)^2 = -9 + 25$ gives
 $x = -9, x = -1$ c) $(x - \frac{5}{2})^2 = 14 + \frac{25}{4}$ gives
 $x = -2, x = 7$
- d) $(x - \frac{3}{2})^2 = 4 + \frac{9}{4}$ gives
 $x = -1, x = 4$ e) $(x-12)^2 = 25 + 12^2$ gives
 $x = -1, x = 25$ f) $(x + \frac{5}{2})^2 = 6 + \frac{25}{4}$ gives
 $x = -6, x = 1$

Problem 5

- a) $k = -25$ b) $k = \pm 6$ c) $k = -\frac{12}{49}$

Problem 6

- a) $(x-2)(x-6)$ b) $(x+9)(x+1)$ c) $(x-9)^2$
- d) $2(x+5)(x-7)$ e) $3(x+11)(x-3)$ f) $5(x+6)^2$

Problem 7

- a) If r is the smallest zero, the polynomial is $x^2 - (2r + 1)x + r(r + 1)$.
b) If r is the second smallest zero, the polynomial is
 $(x - r)^3 - 25(x - r) = x^3 - 3rx^2 + (3r^2 - 25)x - (r - 5)r(r + 5)$.
c) $a(x - r)^2 = ax^2 - 2arx + ar^2$ ($a \neq 0$).

Problem 8

- a) $x = \frac{\sqrt{3}}{2}, x = 9, x = 11$
b) $x = \pm\sqrt{5}, x = 3 \pm \sqrt{5}$
c) $x = 7, x = 1 \pm \sqrt{7}$
d) $x = -1, x = -2, x = -3, x = -a, x = \pm\sqrt{-b}$ for $b \leq 0$

Problem 9

- a) $x = \pm\sqrt{2}, x = \pm 3$ (Hint: Substitute $u = x^2$)
b) $x = -2, x = -\sqrt[3]{5}$
c) $x = \pm\sqrt[25]{2}$

Problem 10

- a) $x = -1$
b) no solutions
c) $x = 0, x = -1.02$