Course paper 1 - EBA1180[1] Mathematics for Data Science

14 – 21 October 2022

The problem set has 2 pages. All 25 subproblems have equal weight. To pass, 60% score is required. You are required to give reasons for all answers.

Your answers should be provided digitally, as one .pdf file. Write by hand with recognisable handwriting. Check that the file is easy to read, pencil writing can result in weak files. For more information: https://portal.bi.no/en/examination/digital-examination/

Problem 1

a) Calculate the sum

$$25\,000 \cdot 1.004^{276} + 25\,000 \cdot 1.004^{274} + 25\,000 \cdot 1.004^{272} + \ldots + 25\,000 \cdot 1.004^{40} + 25\,000 \cdot 1.004^{38}$$

b) Describe a financial situation where this sum is relevant (the important numbers should be interpreted).

Problem 2

Here are some payments at different times:

Year	4	6	9
Payment	A	B	C

Suppose the nominal discount rate is r with continuous compounding.

a) Give an expression for the present value of the cash flow. Calculate the present value of the cash flow if $A = 20$, $B = 30$, $C = 50$ and $r = 10\%$.

b) Give an expression for the future value of the cash flow after 7 years. Calculate the future value of the cash flow after 7 years with the numbers from (a).

c) Suppose $A = 20$, $B = 30$, $C = 50$.
   i) Determine the interest r such that payment A and payment B have the same present value.
   ii) Determine the interest r such that the present value of payment C is larger than the present value of payment B.

d) Suppose $A = 20$, $B = 30$, $C = 50$. Determine the interest r such that the present value of payment B is larger than the present value of each of the two other payments and at the same time the present value of payment A is smaller than than the present value of payment C.

Problem 3

a) Determine the quadratic equation $x^2 + bx + c = 0$ which has the solutions $x = -2 \pm \sqrt{7}$.

b) Determine the third degree equation $x^3 + bx^2 + cx + d = 0$ which has the solutions $x = \pm\sqrt{10}$ and $x = k$.

Problem 4

a) Solve the equation $\sqrt{4x + 9} = x + 1$.

b) Show that the equation $\sqrt{4x + 9} = x + t$ has no solutions if $t > 3.25$.

c) Determine the values of k such that the equation $x^4 + 6x^2 = k$ has solutions.

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Problem 5

We have a function $g(x)$ defined for all numbers and the only things we know about it is that the inequality $g(x) > 0$ has the solutions $x < 3$, the inequality $g(x) < 0$ has the solutions $x > 3$ while $g(3) = 0$. Solve the following inequalities.

a) $(x-1)(x+2)g(x) \geqslant 0$ b) $e^{2-x}\ln(x)g(x) < 0$

c) $\dfrac{(3x-5)g(x)}{x-5} \leqslant g(x)$ d) $\dfrac{g(x)^2+4}{e^{2x}-10} > 0$

Problem 6

We have $f(x) = x^3 + bx^2 + cx - 125$ and $g(x) = x - 5$ where b and c are arbitrary numbers (parameters).

a) Perform the polynomial division $f(x) : g(x)$.
b) Determine when $x - 5$ is a factor in $f(x)$.

Problem 7

a) Suppose the second degree polynomial function $f(x)$ has minimal value -3.5 and the points $P = (75, 4)$ and $Q = (105, 4)$ are on the graph of $f(x)$. Determine the function expression for $f(x)$.
b) Suppose the hyperbola function $g(x)$ has horizontal asymptote $y = 100$ and the y-axis as vertical asymptote. Also assume that $g(5) = 98$. Determine the function expression for $g(x)$.

Problem 8

We have four points in the plane: $A = (6, 4)$, $B = (10, 4)$, $C = (10, 8)$, $D = (6, 8)$.

a) Determine the equation for a circle that passes through the four points.
b) Determine the equation for an ellipse that passes through the four points such that the horizontal semi-axis is greater than the vertical semi-axis.
c) Explain why there are no ellipses which passes through the four points which also passes through the origin $(0, 0)$.

Problem 9

Determine the inverse function $g(x)$. Also determine the domain of definition D_g and the range R_g.

a) $f(x) = 10 + \frac{0{,}2}{x-3}$ with domain of definition $D_f = \langle 3, \infty\rangle$.
b) $f(x) = \ln(10x - x^2)$ with domain of definition $D_f = [1, 5]$.

Problem 10

The following fact can be useful in this problem: If $x > 0$ then $e^x > 1$. Show by an elementary argument (without using differentiation) that the function $f(x) = e^x$ is strictly increasing for all x.