School exam (3h) EBA11802 - Mathematics for Data Science

8 Des. 2022

The exam set has 3 pages. All 12 problems have equal weight. You are required to give reasons for all answers. Grades: A - F which counts for 20% of the final grade in the course.
Support materials permitted: BI-approved exam calculator. Ruler.

Problem 1

In figure 1 you see an ellipse.

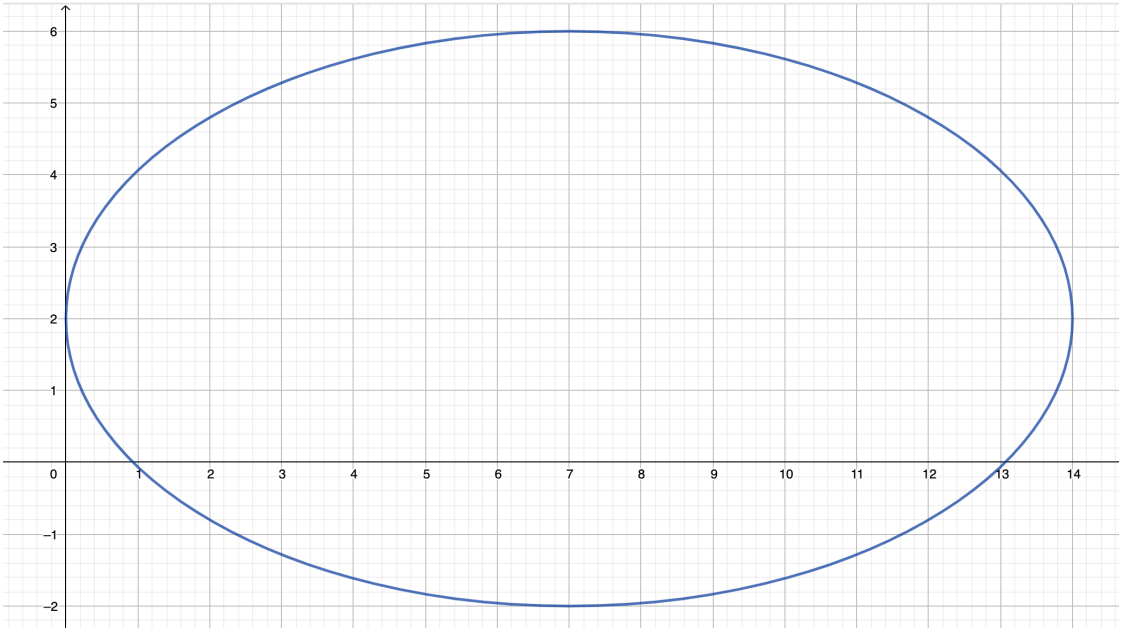

Figure 1: Ellipse

i) Determine the centre and the semi-axes of the ellipse.
ii) Write up the ellipse equation on standard form.

Problem 2

Factorise the polynomial $f(x) = x^3 - 3x^2 - 3x + 1$ into factors of the least possible degree.

Problem 3

Calculate the limit

$$\lim_{x \to 0} \frac{0.5x + 1 - \sqrt{x+1}}{x^2}$$

Problem 4

i) A bank account has 3.6% nominal interest with monthly compounding. Determine the effective interest.
ii) An investment of 15 million is supposed to give a payment of 30 million 6 years from now. Determine the internal rate of return.

Problem 5

Show that the function $f(x) = 0.03x^2 + 5x + 200 + 300e^{0.01x}$ is a cost function.

Problem 6

Let p be the price of a commodity and assume $D(p) = (p + 20)e^{-0.05p}$ with $p > 0$ is the demand function. Let $\varepsilon(p)$ be the momentary price elasticity of the demand function.

i) Calculate $\varepsilon(p)$.
ii) Determine whether the revenue is going up or down if the price is increasing a little from $p = 40$.

Problem 7

Make a sketch of the graph of each of the functions $f(x)$ and $g(x)$ with the given data. Note: You are not supposed to find any algebraic expressions!

i) $f'(x)$ is positive for $x < 8$, negative for $8 < x < 16$ and positive for $x > 16$. Moreover, $f(12) = 10$.
ii) $g''(x)$ is positive for $x < 30$, negative for $30 < x < 60$ and positive for $x > 60$. Moreover, $g'(60) = 0$.

Problem 8

In figure 2 you see the graph of $f'(x)$.

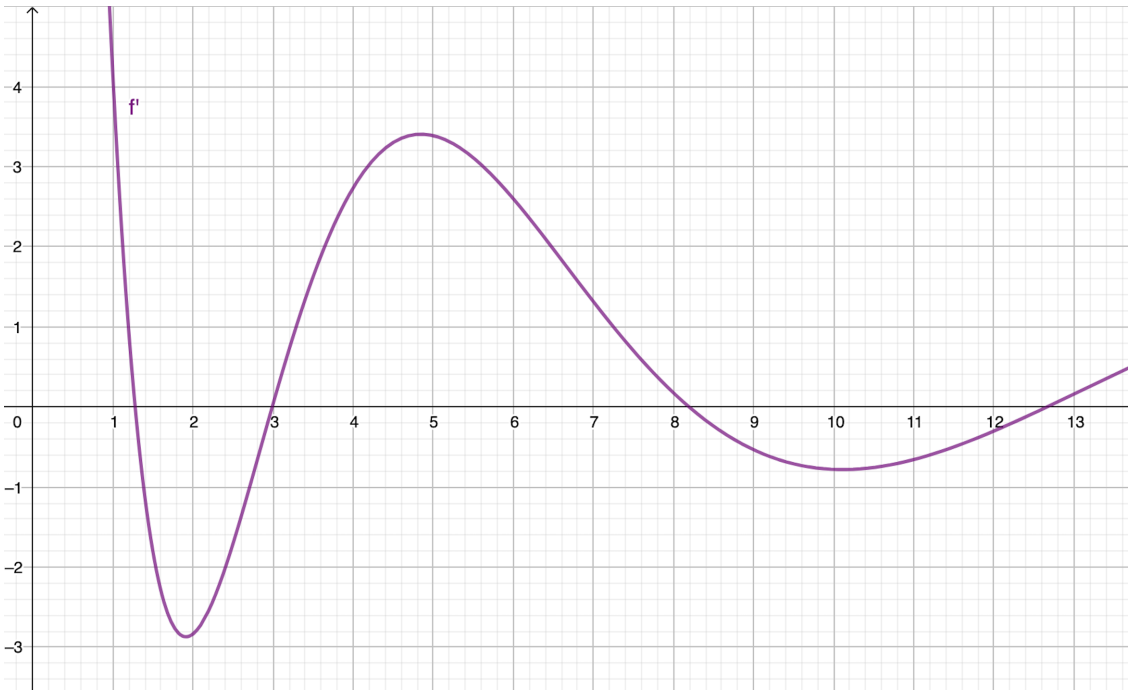

Figure 2: Graph of $f'(x)$

Determine whether the statement is true or false and give a short explanation.

i) $f(x)$ has three stationary points.
ii) $f(4) > f(8)$
iii) $f(x)$ has three inflection points.

Problem 9

We have the function $f(x) = 5\ln(x^2 - 20x + 102)$ with domain of definition $D_f = [0, 25]$.

i) Determine the minimum point and the maximum point of $f(x)$.
ii) Determine the maximum and minimum of $f(x)$.

Problem 10

The function $f(x)$ has $f(10) = 200$, $f'(10) = -3$ and $f''(10) = 1$. Calculate an approximate value for $f(12)$.

Problem 11

We have the function $f(x) = \frac{2022e^x}{e^x+1}$ with domain of definition D_f equal to the whole number line.

i) Determine the domain of definition and the range of the inverse function $g(x)$.
ii) Determine the expression of the inverse function $g(x)$.

Problem 12

i) The price of a commodity changes from a kroner to b kroner. Determine the relative change.
ii) The price of a commodity changes three times with relative changes r_1, r_2 and r_3. After these price changes the commodity costs b kroner. What was its cost before these three price changes?