

School exam (3h) EBA11802 - Mathematics for Data Science

5 May 2023

The exam set has 2 pages. All 12 problems have equal weight. You are required to give reasons for all answers. Grades: A - F which counts for 20% of the final grade in the course.

Support materials permitted: BI-approved exam calculator. Ruler.

Problem 1

Factorise the polynomial $f(x) = x^4 - 7x^2 + 6x$ into factors of the least possible degree.

Problem 2

A hyperbola function $f(x)$ has a horizontal asymptote $y = 100$ and a vertical asymptote $x = 30$. Moreover, $f(40) = 99$.

- i) Determine the expression for $f(x)$.
- ii) Draw a sketch of the graph of $f(x)$ with asymptotes.

Problem 3

Calculate the expression for the differentiated function $f'(x)$ if

- i) $f(x) = x\sqrt{x}$
- ii) $f(x) = \frac{3x-4}{x-1}$
- iii) $f(x) = (2x+3)^{50} + 11$

Problem 4

You are supposed to be paid 2 million every year for n years with the first payment 5 years from now. Suppose the interest is 6% with annual compounding.

- i) Write down the geometric series which gives the present value of the cash flow.
- ii) Use the geometric series to compute the present value of the cash flow if $n = 20$.
- iii) Use the geometric series to compute the present value of the cash flow if the payments continue forever.

Problem 5

- i) Solve the equation $\sqrt{10-x^2} = x - 2$.
- ii) Solve the inequality $\ln(x+2) - \ln(x) \leq 0.1$.

Problem 6

We have a cost function $K(x) = K_0 \cdot e^{0.05x}$ with $x \geq 0$ where K_0 is an undetermined positive number.

- i) Determine the marginal cost function.
- ii) Calculate the cost optimum and the minimal average unit cost.

Problem 7

We have the function $f(x) = 8x^2 - 25 \cdot \ln(x)$.

- i) Determine where $f(x)$ is increasing and where it is decreasing.
- ii) Determine where $f(x)$ is concave and where it is convex.

Problem 8

In figure 1 you see the graph of the function $f(x)$.

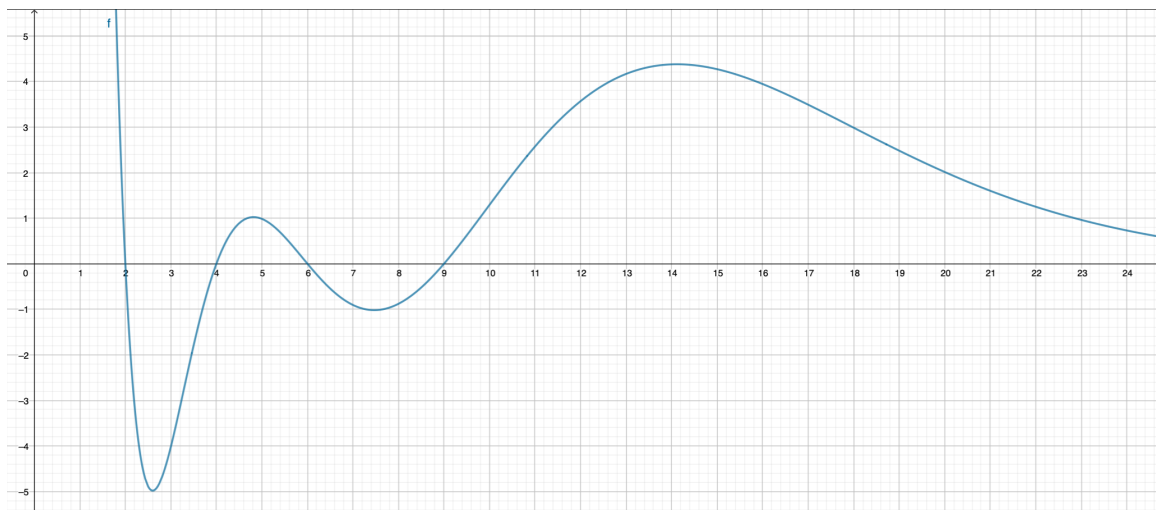


Figure 1: The graph of $f(x)$

Determine whether the statement is true or false and give a short explanation.

- i) $f(x)$ has three stationary points in the interval $[3, 10]$.
- ii) $f(x)$ has three inflection points in the interval $(4, 24]$.
- iii) $f'(x)$ is decreasing in the interval $[12, 24]$.

Problem 9

Let p be the price of a commodity and suppose $D(p) = 30p^{-0.8}$ with $p > 0$ is the demand function. Let $\varepsilon(p)$ be the momentary price elasticity of the demand function.

- i) Calculate $\varepsilon(p)$.
- ii) Determine whether the revenue is going up or down if the price is increasing a little from $p = 20$.

Problem 10

We have the function $f(x) = e^{-0.02x} + 100$ with domain of definition $D_f = [0, \infty)$. Let $g(x)$ be the inverse function of $f(x)$.

- i) Determine the expression of $g(x)$.
- ii) Determine the domain of definition D_g and the range R_g of $g(x)$.

Problem 11

- i) A bank account has 7.2% nominal interest and continuous compounding. Determine the effective interest.
- ii) Suppose A and B are two undetermined positive numbers. An investment of A million is supposed to give a payment of B million 5 years from now. Determine the internal rate of return for the cash flow.

Problem 12

We have a curve implicitly given by the equation $x^3 - 4xy + y^2 = -2$.

- i) Find an expression for y' in terms of y and x .
- ii) Determine the expression for the tangent of the curve at the point $(1, 3)$.