School exam (3h) EBA11805 - Mathematics for Data Science

5 December 2025

The exam set has 3 pages. All 12 problems have equal weight. You are required to give reasons for all answers. Grades: A - F which counts for 40% of the final grade in the course.

Problem 1

i) Use polynomial division to calculate $(x^3+11x^2+40x+50):(x+5)$ with remainder.

ii) Factorise $x^3+11x^2+40x+50$.

Problem 2

i) Calculate the sum of the geometric series:

$$\frac{20\,000}{1.03^2}+\frac{20\,000}{1.03^3}+\cdots+\frac{20\,000}{1.03^{24}}+\frac{20\,000}{1.03^{25}}$$

ii) Describe a financial situation where this sum is relevant (the important numbers should be interpreted).

Problem 3

In figure 1 you see the graph of the function $f(x)$.

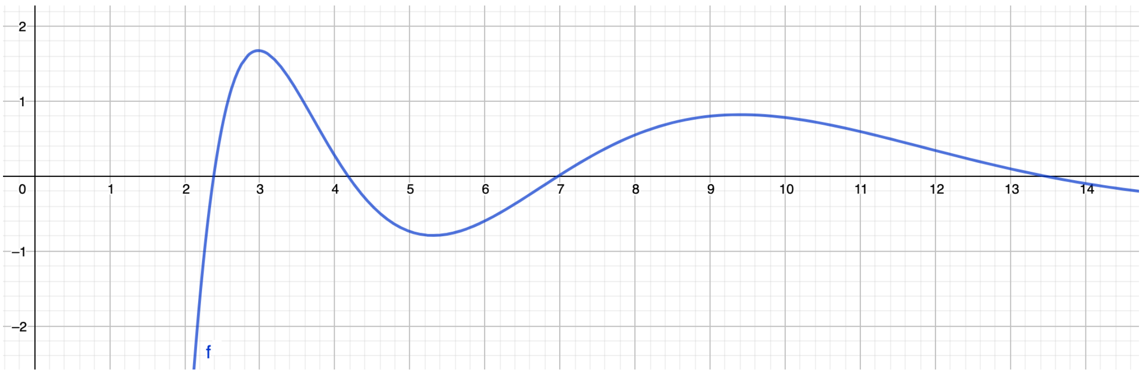

Figure 1: The graph of $f(x)$

Determine whether the statement is true or false. Give a short explanation.

i) $f'(3)>f'(8)$.

ii) $f(x)$ has three stationary points in the interval $[2,8]$.

iii) $f(x)$ has two inflection points in the interval $[2,10]$.

Problem 4

The hyperbola function $f(x)$ has $x=30$ as vertical asymptote and $y=50$ as horizontal asymptote. Moreover, $f(33)=48$. Determine the expression of the hyperbola function.

Problem 5

The graphs of the functions $f(x)$ and $g(x)$ are given in figure 2.

i) Use the figure to solve the inequality $f(x) - g(x) > 0$.

ii) Use the figure to solve the inequality $\frac{f(x)}{g(x)} \leqslant 0$.

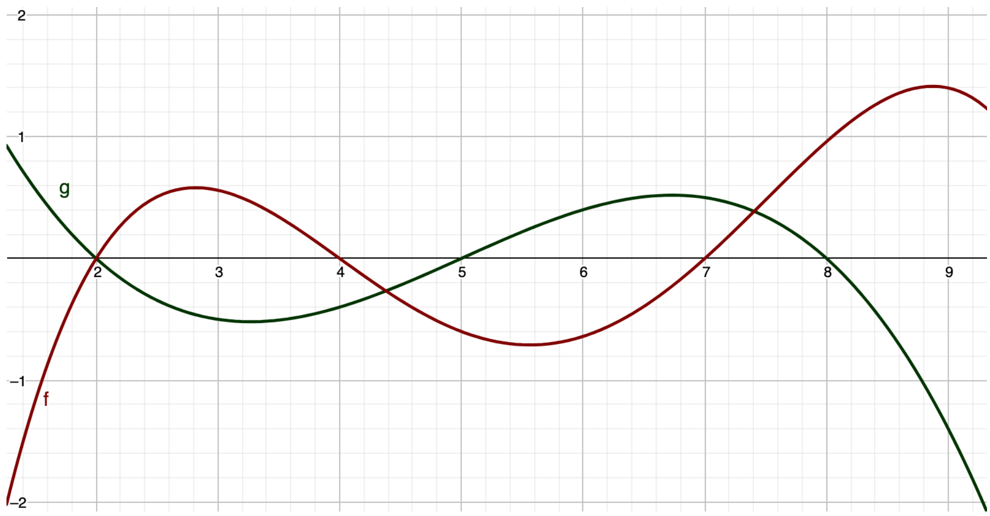

Figure 2: The graphs of $f(x)$ and $g(x)$

Problem 6

i) Calculate how much you have to deposit today to have 800 000 on the account 12 years from now if the interest is 4.6%.

ii) You would like the balance on the account 6 years from now to be 800 000 with the same deposit as in (i). Calculate what the interest has to be.

Problem 7

An ellipse has centre $(3, 5)$, the horizontal semi-axis is 2 and the vertical semi-axis is 4.

i) Write up the equation for the ellipse on standard form.

ii) Make a drawing of the ellipse.

Problem 8

Make a drawing of the graph of a function $f(x)$ with domain of definition $D_f = [5, 30]$ which satisfies the following three requirements:

1) $f(20) = 100$.

2) $f'(x)$ is negative for all x.

3) $f''(x) > 0$ for $x < 15$ and $f''(x) < 0$ for $x > 15$.

Problem 9

Let p be the prize of a commodity and suppose $D(p) = 150e^{-0.01p^2}$ with $p \geqslant 0$ is the demand function. Suppose $\varepsilon(p)$ is the momentary price elasticity of the demand function.

i) Calculate $\varepsilon(p)$.

ii) Determine the price p such that the demand is elastic.

iii) Determine whether the revenue is increasing or decreasing if the price increases a little from $p = 10$.

Problem 10

A curve is implicitly defined by the equation $e^y = xy + 2025$. Use implicit differentiation to express y' in terms of y and x.

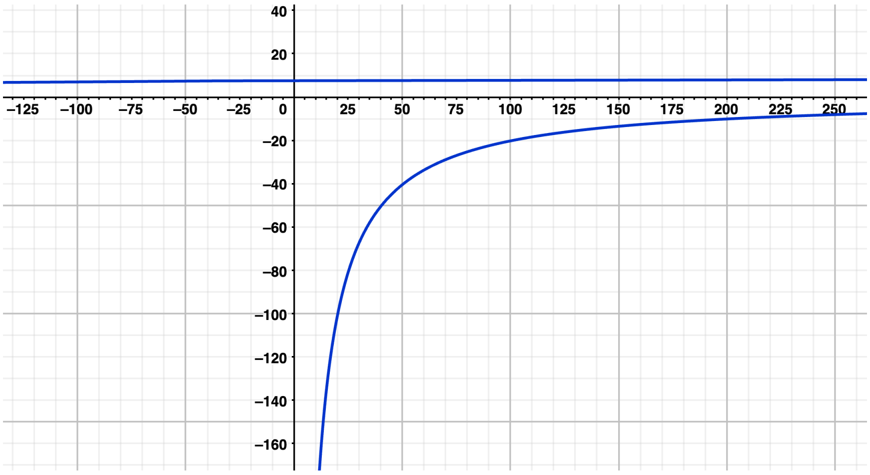

Figure 3: Implicitly defined curve

Problem 11

We have the function

$$f(x) = \ln\left(\frac{50e^{0.03x} + 1}{120e^{0.03x} + 2025}\right)$$

Determine possible horizontal and vertical asymptotes of $f(x)$.

Problem 12

We have the function $f(x) = e^x \ln(x+1)$. Find the degree 3 Taylor polynomial $P_3(x)$ of $f(x)$ about 0.