Exam	EBA 1180 Mathematics for Data Science
Date	December 18th 2025 at 0900 - 1400

The exam consists of 15 sub-questions with equal weight. All answers must be justified.

Question 1.
Consider the linear system $A\mathbf{x} = \mathbf{0}$, where the matrix A is given by

$$A = \begin{pmatrix} 1 & 3 & 1 & 2 & 4 \\ 3 & 4 & 8 & 4 & 5 \\ 2 & 6 & 2 & 5 & 4 \end{pmatrix}$$

(a) Solve the linear system $A\mathbf{x} = \mathbf{0}$. How many degrees of freedom are there?
(b) How many solutions does the linear system $A\mathbf{x} = \mathbf{b}$ have for different choices of the vector $\mathbf{b}$?

Question 2.
Compute the following integrals:

a) $\int_1^4 \frac{\sqrt{x}}{x^2}\,\mathrm{d}x$ b) $\int_0^5 \frac{7}{x^2 - 5x - 6}\,\mathrm{d}x$ c) $\int_1^4 \frac{\ln x}{2\sqrt{x}}\,\mathrm{d}x$ d) $\int \frac{1}{e^x + 1}\,\mathrm{d}x$

Question 3.
You are to invest in a portfolio of stocks and can choose a combination of the firms A, B, C with price $p_A = 30$, $p_B = 80$ and $p_C = 200$ NOK per share at the time of investment. We assume that at a specific time in the future, one out of three scenarios will happen. The prices of the shares in these scenarios are given in the table below. We write x, y, z for the number of shares you buy in

	Price A	Price B	Price C
Purchase price	30	80	200
Scenario 1	10	120	100
Scenario 2	80	20	350
Scenario 3	60	100	250

each of the three firms, and we assume that x,y,z can be any real numbers. Hence, we allow buying a negative number of shares (short selling), and the number of shares does not have to be an integer.

(a) Find the payoffs R_1, R_2 and R_3 of the portfolio in the three scenarios expressed via x,y,z.
(b) Is it possible to choose the portfolio such that $R_1 = 20.000$ NOK, $R_2 = 10.000$ NOK and $R_3 = 70.000$ NOK? If so, determine how many shares we much purchase in firms A, B and C, as well as the initial purchase price (cost price) of this portfolio.

Question 4.
Consider the function f given by $f(x,y) = xy^3 - x^3y + 4x^2y - 4xy$.

(a) Compute the partial derivatives of f and the Hessian matrix $H(f)$.
(b) Find the stationary points of f with $x \neq 0$ and $y \neq 0$.
(c) Classify the stationary points with $x \neq 0$ and $y \neq 0$.
(d) Determine whether f has a global maximum or minimum value.

Question 5.
Consider the Lagrange problem $\max / \min f(x,y) = y^2 - x^2 + 4x - 4$ when $x = 2y^2$.

(a) Write down the three Lagrange conditions, and find all points $(x,y;\lambda)$ that satisfy these.
(b) Sketch the curve $x = 2y^2$, and draw the points (if any) that you found in (a).
(c) Determine the maximum or minimum values of the problem, if they exist.

Formula Sheet

Financial mathematics

Geometric series.
A finite geometric series has sum

$$S_n = a_1 \cdot \frac{1-k^n}{1-k} = a_1 \cdot \frac{k^n-1}{k-1}$$

and an infinite geometric series has sum

$$S = a_1 \cdot \frac{1}{1-k} \quad \text{when } |k| < 1$$

Present values.
The present value K_0 of a payment K is given by

$$K_0 = \frac{K_n}{(1+r)^n} \quad \text{and} \quad K_0 = \frac{K_n}{e^{rn}}$$

using discrete and continuous compounding.

Integration

Integration techniques.

a) Integration by parts:

$$\int u'v \,\mathrm{d}x = uv - \int uv' \,\mathrm{d}x$$

b) Substitution:

$$\int f(u)u' \,\mathrm{d}x = \int f(u) \,\mathrm{d}u$$

c) Partial fractions:

$$\int \frac{px+q}{(x-a)(x-b)} \,\mathrm{d}x = \int \left(\frac{A}{x-a} + \frac{B}{x-b} \right) \mathrm{d}x$$

Area.
The area of the region bounded by $a \le x \le b$ and $f(x) \le y \le g(x)$ is given by

$$A = \int_a^b \left(g(x) - f(x) \right) \mathrm{d}x$$

Linear algebra

Cramer's rule.
A linear system $A\mathbf{x} = \mathbf{b}$ where $|A| \neq 0$ has a unique solution given by

$$x_1 = \frac{|A_1(\mathbf{b})|}{|A|} \quad x_2 = \frac{|A_2(\mathbf{b})|}{|A|} \quad \dots \quad x_n = \frac{|A_n(\mathbf{b})|}{|A|}$$

where $A_i(\mathbf{b})$ is the matrix obtained by replacing column i of A by $\mathbf{b}$.

Functions of two variables

Second derivative test.
A stationary point (x^*, y^*) of the function $f(x,y)$ is a

a) local minimum if $A > 0$ and $AC - B^2 > 0$
b) local maximum if $A < 0$ and $AC - B^2 > 0$
c) saddle point if $AC - B^2 < 0$

when $H(f)(x^*, y^*) = \left(\begin{smallmatrix} A & B \\ B & C \end{smallmatrix}\right)$.

Level curves.
The slope $y' = \mathrm{d}y/\mathrm{d}x$ of the tangent line to the level curve $f(x,y) = c$ is given by

$$y' = -\frac{f'_x}{f'_y}$$

Method of Lagrange multipliers.
The Lagrange conditions for the problem

$$\max / \min f(x,y) \text{ when } g(x,y) = a$$

is given by

$$\mathcal{L}'_x = 0, \ \mathcal{L}'_y = 0, \ g(x,y) = a$$

An admissible point has degenerated constraint if

$$g'_x = 0, \ g'_y = 0$$