Course paper 1 - EBA2911[1] **Mathematics for Business Analytics**

15 Oct. – 22 Oct. 2021

The problem set has 3 pages. All 25 subproblems have equal weight. To pass 60% score is required.

You are required to give reasons for all answers.

Your answers should be provided digitally, as a .pdf file. Write by hand with recognisable handwriting. Check that the file is easy to read, pencil writing can result in weak files. For more information, see: https://portal.bi.no/en/examination/digital-examination/

Problem 1

a) i) Calculate the sum

$$6\,000 \cdot 1.0025^{96} + 6\,000 \cdot 1.0025^{95} + 6\,000 \cdot 1.0025^{94} + \ldots + 6\,000 \cdot 1.0025^{26} + 6\,000 \cdot 1.0025^{25}$$

ii) Describe a financial situation where the sum is used (the important numbers should be interpreted).

b) Describe a financial situation where the sum

$$1\,000\,000 - \frac{15\,000}{e^{48r}} - \frac{15\,000}{e^{49r}} - \frac{15\,000}{e^{50r}} - \cdots - \frac{15\,000}{e^{167r}} = 0$$

is used (the important numbers should be interpreted).

Problem 2

Kåre wants to borrow money to buy a flat. He reckons he can afford to pay 10 000 each month with first payment 5 years from now. Let r denote the nominal interest. Suppose it is monthly compounding.

a) Assume a down payment period of 30 years.
   i) Determine an expression for how much Kåre can borrow (the mortgage).
   ii) Calculate the mortgage if $r = 3\%$ and if $r = 6\%$.

b) Calculate the mortgage if $r = 6\%$ and Kåre pays without end date (forever).

Problem 3

Hege considers an investment proposition from the building contractor *Tall Cranes* given by the cash flow

Year	0	2	5	7	8
Payment	−120	−170	100	200	250

Suppose the discount rate is 14%.

a) i) Calculate the present value of the cash flow.
   ii) Calculate the future value of the cash flow after 6 years.

b) For the investment to have 14% as internal rate of return (IRR) Hege suggests an extra payment after 6 years. Determine this payment.

c) *Tall Cranes* say they can accept Hege's proposal if they also can reduce the back payment after 8 years from 250 to 200. Determine how much the second payment (of 170) has to be changed so that the IRR of the new cash flow (with *Tall Crane*'s proposal) becomes 14%.

[1] Exam code EBA29101

Problem 4

Solve the equations.

a) $(e^x - 2)(\ln(x) - 3)(4x^2 + 5x^3) = 0$

b) $x^8 - 12x^4 = 64$

c) $\sqrt{2x-5} = 2 - x$

Problem 5

Solve the inequalities.

a) $\dfrac{4x+9}{x^2+2x+3} \geqslant 2$

b) $\ln(x) + \ln(x-2) - \ln(x-3) \leqslant \ln(8)$

c) $\dfrac{\ln(x)+2}{e^x - 4} \geqslant 0$

Problem 6

We have $f(x) = x^4 + 2x^3 - 28x^2 + 46x - 21$ and $g(x) = x - t$ where t is an arbitrary number (a parameter).

a) Suppose $t = 1$. Perform the polynomial division $f(x) : g(x)$.
b) Determine the remainder of the polynomial division $f(x) : g(x)$ for all t.
c) Determine all values of t such that the polynomial division $f(x) : g(x)$ has remainder 0.

Problem 7

Figure 1 shows a part of a parabola which is the graph of a second degree polynomial function $f(x)$. Determine the zeros (roots) of $f(x)$.

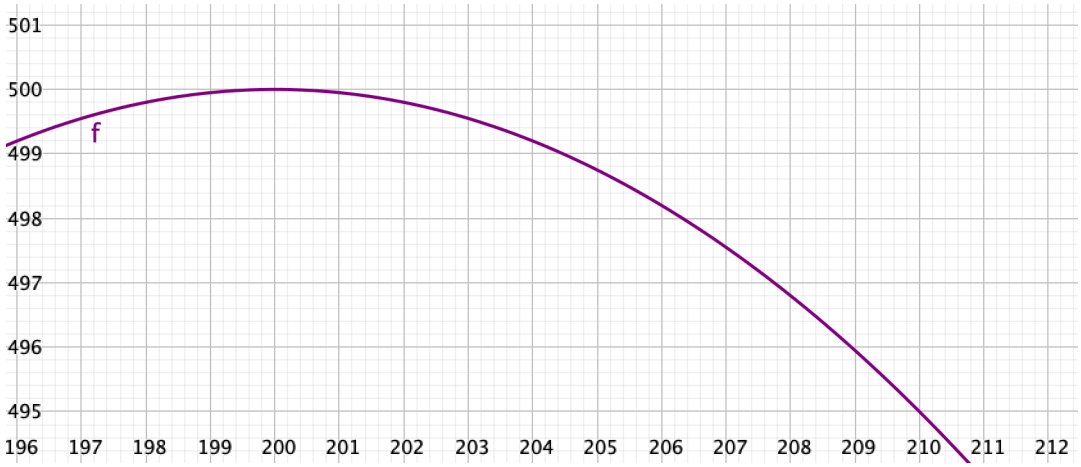

Figure 1: Parabola

Problem 8

a) Suppose the second degree polynomial function $f(x)$ has maximum value 100 and the points $P = (8, 90)$ og $Q = (12, 90)$ are on the graph of $f(x)$. Determine the expression for $f(x)$.
b) Suppose $P = (8, 90)$ og $Q = (12, 90)$ are on the graph of the second degree polynomial function $g(x)$. Show that the expression for $g(x)$ can be written as

$$g(x) = \frac{90-d}{4}(x-10)^2 + d$$

where d gives the maximal value (or the minimal value) of $g(x)$.
c) Determine the values of d such that $g(x)$ has a maximum point.

Problem 9

Figure 2 shows a part of an ellipse. Determine the equation of the ellipse. In particular give the semi-axes and the centre of the ellipse.

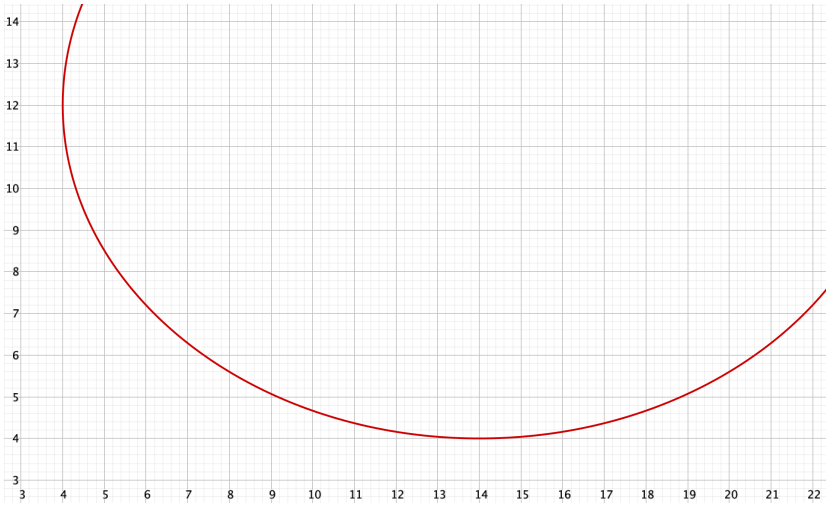

Figure 2: Ellipse

Problem 10

A part of the graph of the hyperbola function $f(x)$ is shown in figure 3.

a) Determine the expression of $f(x)$.
b) Here is another hyperbola function: $g(x) = 16 - \frac{3}{(x-12)}$. Determine the intersection points of the two parabolas.

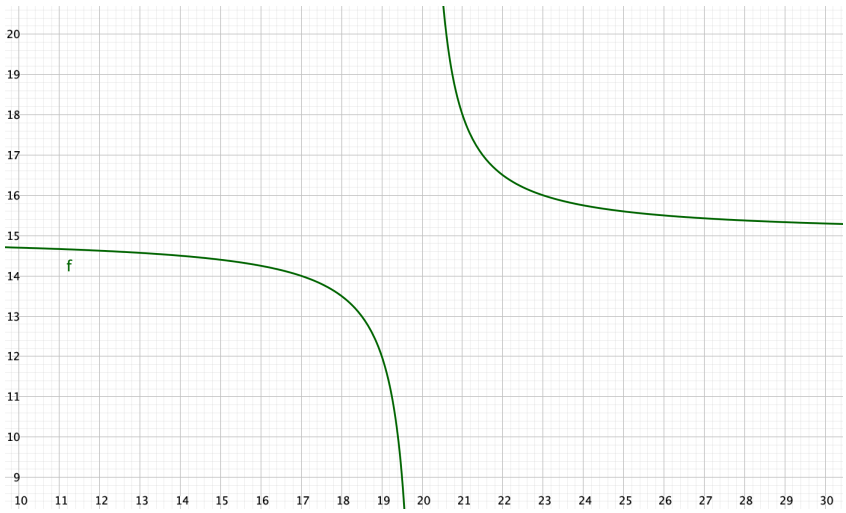

Figure 3: Hyperbola

Problem 11

Determine the inverse function $g(x)$. Also determine the domain of definition D_g and the range R_g.

a) $f(x) = \sqrt{x-1} + 3$ with domain of definition $D_f = [1, 26]$.
b) $f(x) = e^{-0.1x+2} + 5$ with domain of definition $D_f = [10, \infty\rangle$.