Multiple choice 1 in EBA2910[1] - Mathematics for Business Analytics
10 December 2021

SOLUTIONS

Problem	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Right answer	C	B	D	C	B	D	B	A	A	A	C	B	C	D	A

Problem 1

We have $C(0) = 1600 > 0$, $C'(x) = 2x + 20 > 0$ for $x > 0$ and $C''(x) = 2 > 0$ for all x. Hence the cost optimum is the solution of the equation $C'(x) = \frac{C(x)}{x}$. Multiply both sides with x and get $2x^2 + 20x = x^2 + 20x + 1600$, i.e. $x^2 = 1600$, i.e. $\underline{x = 40}$ (because $x > 0$). This is not one of the answers so we calculate the optimal unit cost as $C'(40) = 2 \cdot 40 + 20 = \underline{100}$.

Right answer is C.

Problem 2

The first deposit will remain in the account for $18 \cdot 12 = 216$ interest periods with period interest $\frac{3.6\%}{12} = 0.003$ which gives the future value 18 years from now as $5000 \cdot 1.003^{216}$. The second deposit will remain in the account for 215 interest periods which gives the future value $5000 \cdot 1.003^{215}$, and so on until the last deposit which remains in the account for $3 \cdot 12 + 1 = 37$ months which gives the future value $5000 \cdot 1.003^{37}$. The sum of these future values gives the account balance 18 years from now.

The right answer is B.

Problem 3

(A) $f'(x) = \frac{1}{x}$ så $f'(0.5) = \frac{1}{0.5} = 2$
(B) $f'(x) = \frac{1 \cdot (1-x) - x(-1)}{(1-x)^2} = \frac{1}{(1-x)^2}$ so the equation $f'(x) = 0$ has no solutions.
(C) $f'(x) = e^x + xe^x$ så $f'(-1) = e^{-1} + (-1)e^{-1} = 0$
(D) $f'(x) = \frac{1}{2\sqrt{x}}$ so $f'(0.25) = \frac{1}{2\sqrt{0.25}} = \frac{1}{2 \cdot 0.5} = 1$.

The right answer is D.

Problem 4

The graph of the hyperbola function is symmetric about the intersection point $(30, 80)$ of the asymptotes. Since 40 is 10 more than 30 and 20 is 10 less, the points $(40, f(40))$ and $(20, 90)$ are on each side of and equally distanced from the symmetry point $(30, 80)$, i.e. $f(40) = 70$. Alternatively we can find the expression $f(x) = 80 - \frac{100}{x-30}$ and calculate $f(40) = 70$.

The right answer is C.

Problem 5

The present value of the cash flow is

$$\frac{12}{1.15^3} + \frac{21}{1.15^4} + \frac{26}{1.15^5} = 32.82$$

This also has to be the value of K for the internal rate of return to be 15%.

The right answer is B.

[1] Exam code EBA29102

Problem 6

(A) We read off the graph that $f(2) \approx 2.1$ while $f(5.5) \approx 1.85$ so $f(2) > f(5.5)$.
(B) We estimate the slope of the tangents to $f'(2) \approx -0.8$ and $f'(5) \approx -1.75$ so $f'(2) > f'(5)$.
(C) We see from the graph that $f(x)$ is concave in the interval $[4\,,5]$ and $f'(x)$ is hence decreasing in the interval $[4\,,5]$.
(D) It seems that $f(x)$ has an inflection point approximately at $x = 3.4$. Hence $f''(x)$ changes sign in the interval $[3\,,4]$.

The right answer is D.

Problem 7

$f'(x) = e^x - 2$ og $f''(x) = e^x$. Hence $f''(x) > 0$ for all x and $f(x)$ is convex.

The right answer is B.

Problem 8

The domain of definition D_g for g equals the range R_f of f. Because $f(x)$ is decreasing in the interval $[0,3]$, the maximum value is $f(0) = 100 - 5^2 = 75$ while the minimum value is $f(3) = 100 - 8^2 = 36$. By the intermediate value theorem the continuous function $f(x)$ achieves all values inbetween when x runs through $D_f = [0,3]$. Hence $D_g = R_f = [36,75]$.

The right answer is A.

Problem 9

We subtract $5x$ on each side of the inequality and get $xe^x - 5x \leqslant 0$. Factor out x and get $x(e^x - 5) \leqslant 0$. By a sign diagram $0 \leqslant x \leqslant \ln(5)$.

The right answer is A.

Problem 10

With n payments the present value becomes

$$\frac{15\,000}{1.004^{240}} + \frac{15\,000}{1.004^{241}} + \cdots + \frac{15\,000}{1.004^{n+239}}$$

If we read this geometric series from left to right we get the sum

$$\frac{15\,000}{1.004^{240}} \cdot \frac{\left(\frac{1}{1.004}\right)^n - 1}{\left(\frac{1}{1.004}\right) - 1}$$

If n grows without limits $\left(\frac{1}{1.004}\right)^n = \frac{1}{1.004^n}$ approaches 0 more and more. The present value of the regular cash flow without end date can thus be interpreted as

$$\frac{15\,000}{1.004^{240}} \cdot \frac{-1}{\left(\frac{1}{1.004}\right) - 1} = \frac{15\,000}{1.004^{240}} \cdot \frac{1}{1 - \left(\frac{1}{1.004}\right)} = \frac{15\,000}{1.004^{240}} \cdot \frac{1.004}{0.004} = 1\,444\,354.86$$

The right answer is A.

Problem 11

By the chain rule we have $f'(x) = g'(u) \cdot u'(x)$ where $u = u(x)$. From the tables we get $u(35) = 3$, $g'(3) = 4$ and $u'(35) = 0.5$ which gives $f'(35) = 0.5 \cdot 4 = 2$.

The right answer is C.

Problem 12

The stationary points for $f(x)$ are the solutions of the equation $f'(x) = 0$.

(A) $e^x - 10 = 0$ gives the solution $x = \ln(10)$ while $x^2 + 5 = 0$ has no solutions.

(B) $2x-5=0$ gives the solution $x=2.5$ while $\ln(x^2-14x+50)=0$ if and only if $x^2-14x+50=1$, i.e. $(x-7)^2=1-50+49=0$, i.e. $x=7$.
(C) $\ln(x)-2021=0$ has one solution $x=e^{2021}$.
(D) $x\ln(x)-\ln x=0$ corresponds to $(x-1)\ln x=0$. But $x-1=0$ and $\ln(x)=0$ have the same solution $x=1$.

The right answer is B.

Problem 13

(A) $f(x)$ has inflection points where $f''(x)$ changes sign, i.e. $x\approx 3$ and $x\approx 10$ which both are contained in the interval $[2,16]$.
(B) $f'(x)$ is strictly decreasing in the interval $[8,10]$ because $f''(x)<0$ for $x\in[8,10\rangle$ while $f'(x)$ is strictly increasing in the interval $[10,16]$ because $f''(x)>0$ for $x\in\langle 10,16]$.
(C) $f'(x)$ is strictly increasing in the interval $[14,32]$ because $f''(x)>0$ for $x\in[14,32]$. Hence $f'(14)$ is the minimal value.
(D) $f(x)$ is convex in the interval $[18,24]$ because $f''(x)>0$ for $x\in[18,24]$.

The right answer is C.

Problem 14

(A) By l'Hôpital's rule $\lim\limits_{x\to 0}\dfrac{3x}{e^{ax}-1}=\lim\limits_{x\to 0}\dfrac{3}{ae^{ax}}=\dfrac{3}{ae^0}=\dfrac{3}{a}$ and the equation $\dfrac{3}{a}=6$ gives $a=0.5$.

(B) By l'Hôpital's rule $\lim\limits_{x\to 0}\dfrac{\ln(ax+1)}{2x}=\lim\limits_{x\to 0}\dfrac{\frac{a}{ax+1}}{2}=\dfrac{a}{2}$ and the equation $\dfrac{a}{2}=5$ gives $a=10$.

(C) $\lim\limits_{x\to 3}\dfrac{4x-12}{\sqrt{x+1}}+ax=\dfrac{4\cdot 3-12}{\sqrt{3+1}}+a\cdot 3=3a$ and the equation $3a=12$ gives $a=4$.

(D) If $a\neq 1$ we get $\lim\limits_{x\to 1}\dfrac{\ln(x)-x+1}{x^2-2x+a}=\dfrac{\ln(1)-1+1}{1^2-2\cdot 1+a}=0$. But if $a=1$ we can use l'Hôpital's rule twice: $\lim\limits_{x\to 1}\dfrac{\ln(x)-x+1}{x^2-2x+1}=\lim\limits_{x\to 1}\dfrac{\frac{1}{x}-1}{2x-2}=\lim\limits_{x\to 1}\dfrac{\frac{-1}{x^2}}{2}=\dfrac{\frac{-1}{1^2}}{2}=-0.5$.

The right answer is D.

Problem 15

If $x=7$ we get the equation $49-21y+y^2=-5$, i.e. $\left(y-\frac{21}{2}\right)^2=-5-49+\left(\frac{21}{2}\right)^2=\frac{441-216}{4}=\frac{225}{4}$. That gives $y=\frac{21}{2}\pm\frac{15}{2}$, i.e. the two points $(7,3)$ and $(7,18)$.

We differentiate both sides of the equation with respect to x. By using the product rule on xy and the chain rule for y^2 we get the equation $2x-3y-3xy'+2yy'=0$, i.e. $(2y-3x)y'=3y-2x$, i.e.

$$y'=\frac{3y-2x}{2y-3x}$$

The tangent of the curve at $(7,3)$ has slope $y'=\frac{3\cdot 3-2\cdot 7}{2\cdot 3-3\cdot 7}=\frac{-5}{-15}=\frac{1}{3}$.

The tangent of the curve at $(7,18)$ has slope $y'=\frac{3\cdot 18-2\cdot 7}{2\cdot 18-3\cdot 7}=\frac{40}{15}=\frac{8}{3}$.

The product of the slopes is $\frac{1}{3}\cdot\frac{8}{3}=\frac{8}{9}$.

The right answer is A.

Here is the curve with the tangent lines.

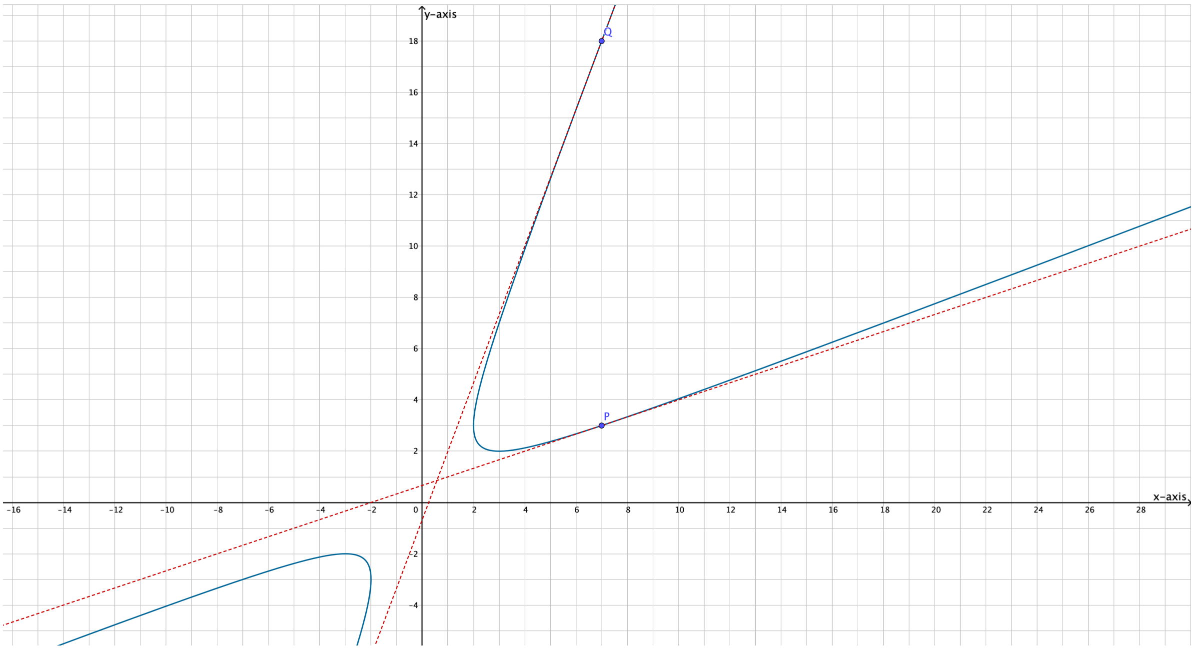

Figure 1: $x^2 - 3xy + y^2 = -5$ with tangent lines