Multiple choice exam EBA29102 - Mathematics for Business Analytics

10 May 2022

SOLUTIONS

Problem	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Right answer	D	C	C	A	A	D	B	C	C	A	D	C	D	C	C

Problem 1

We complete the square and get $x^2+2x+2=(x+1)^2+1 \geqslant 1$ so the equation $x^2+2x+2=0$ has no solutions.

The right answer is D.

Problem 2

With $f(x)=xe^x$ we use the product rule and get $f'(x)=e^x+xe^x=(x+1)e^x$. The equation $f'(x)=0$ has the solution $x=-1$.

The right answer is C.

Problem 3

The interest formula gives $S=500\,000\cdot e^{8r}$. We get $500\,000\cdot e^{8\cdot 1.9}=582\,080$.

The right answer is C.

Problem 4

Because $f(x)$ is continuous and defined on the closed interval $D_f=[0\,,4]$ there are both global maximum points and minimum points for the function. They are either end points or stationary points in D_f. We have $f'(x)=-3x^2+12x-9=-3(x^2-4x+3)=-3(x-1)(x-3)$ so the stationary points are $x=1$ and $x=3$ which both are contained in D_f. We calculate $f(0)=10$, $f(1)=6$, $f(3)=10$ and $f(4)=6$.

The right answer is A.

Problem 5

The sum of the present values of the $25\cdot 12=300$ payments is

$$\frac{9\,000}{1.002^{36}}+\frac{9\,000}{1.002^{37}}+\cdots+\frac{9\,000}{1.002^{335}}=\frac{9\,000}{1.002^{335}}\cdot\frac{1.002^{300}-1}{0.002}$$

where we use the formula for the sum of a geometric series with $a_1=\frac{9\,000}{1.002^{335}}$, $k=1.002$ og $n=300$.

The right answer is A.

Problem 6

We read off $f(1)\approx 1.6$ and $f(3)\approx 0.95$. Moreover, $f'(x)$ gives the slope of the tangents of the graph of $f(x)$. We see that $f'(2)<0$ while $f'(4)>0$. Because the graph seems to be concave for x in the interval $[4\,,5]$ we have $f''(x)<0$ which means $f'(x)$ is decreasing in this interval. Hence the claims (A)-(C) are correct.

The right answer is D.

Problem 7

To solve the inequality $\frac{x+5}{x-3} < x$ we subtract x from both sides and make a common denominator for the fractions:

$$\frac{x+5}{x-3} - x < 0 \qquad \text{i.e.} \qquad \frac{x+5-x(x-3)}{x-3} < 0 \qquad \text{i.e.} \qquad \frac{-x^2+4x+5}{x-3} < 0$$

We have $-x^2+4x+5 = -(x^2-4x-5) = -(x+1)(x-5)$ which gives the inequality in factorised form:

$$\frac{-(x+1)(x-5)}{x-3} < 0 \qquad \text{i.e.} \qquad \frac{(x+1)(x-5)}{x-3} > 0$$

By a sign diagram we get the the solutions $-1 < x < 3$ or $x > 5$.

The right answer is B.

Problem 8

We have $D'(p) = -5$ and then

$$\varepsilon(p) = \frac{D'(p) \cdot p}{D(p)} = \frac{-5p}{180-5p} = \frac{p}{p-36}$$

The demamnd is inelastic if $\varepsilon(p) > -1$ i.e. $\frac{p}{p-36} > -1$. To solve the inequality we add 1 on both sides and find a common denominator for the fractions:

$$\frac{p}{p-36} + 1 > 0 \qquad \text{i.e.} \qquad \frac{2p-36}{p-36} > 0$$

Because $0 < p < 36$, the denominator is negative. The fraction is therefore positive exactly for those p such that $2p - 36 < 0$, i.e. $p < 18$.

The right answer is C.

Problem 9

In the standard form $f(x) = a(x-s)^2 + d$ we have $s = 11$ and $d = 30$, i.e. $f(x) = a(x-11)^2 + 30$. From the graph we also observe $f(16) = 25$, i.e. $a(16-11)^2 + 30 = 25$ i.e. $a = -0.2$ and $f(x) = -0.2(x-11)^2 + 30$. Then $f'(x) = -0.4(x-11)$ and $f'(21) = -0.4(21-11) = -4$.

The right answer is C.

Problem 10

In the standard form $f(x) = c + \frac{a}{x-b}$ we have $b = 20$ and $c = 50$, i.e. $f(x) = 50 + \frac{a}{x-20}$. Then $f(30) = 49$ gives the equation $50 + \frac{a}{30-20} = 49$ i.e. $a = -10$ and $f(x) = 50 - \frac{10}{x-20}$. We differentiate $f(x)$ and get $f'(x) = \frac{10}{(x-20)^2}$ which will be greater than 0 for all $x \neq 20$.

The right answer is A.

Problem 11

We try to find a line through the origin which also is a tangent to the graph of $K(x)$. Then the x-value of the tangential point is the cost optimum (the production volume which gives the lowest unit cost). It seems to be close to $x = 60$.

The right answer is D.

Problem 12

We always have that $D_g = R_f$, the range of $f(x)$. We have $f'(x) = -\frac{1}{x}$ which is negative in the domain D_f. Hence $f(x)$ is a (strictly) decreasing function. Because $f(e^5) = 0$ while $f(x) \longrightarrow \infty$ if $x \longrightarrow 0^+$ we get $D_g = R_f = [0, \rightarrow\rangle$.

The right answer is C.

Problem 13

$f(x)$ is concave in those intervals where $f''(x) \leqslant 0$, i.e. where $e^x \leqslant 2$ i.e. for $x \leqslant \ln(2)$. $f(x)$ is convex in those intervals where $f''(x) \geqslant 0$, i.e. where $e^x \geqslant 2$ i.e. for $x \geqslant \ln(2)$.

The right answer is D.

Problem 14

We differentiate $f(x)$ three times to find $P_3(x)$: $f'(x) = x^{-1}$, $f''(x) = -x^{-2}$ and $f'''(x) = 2x^{-3}$. It gives $f(1) = 0$, $f'(1) = 1$, $f''(1) = -1$ and $f'''(1) = 2$. Then $P_3(x) = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$. In particular, $P_3(2) = \frac{5}{6} \approx 0.83$.

The right answer is C.

Problem 15

We differentiate (implicitly) both sides of the equation with respect to x and get $\frac{10yy'}{y^2+1} = 1$. We solve for y' and get

$$y' = \frac{y^2+1}{10y}$$

If $y = 1$ we get $x = 5\ln(1^2+1) + 10 = 5\ln(2) + 10$ and $y' = \frac{1^2+1}{10\cdot 1} = 0.2$.

The right answer is C.