EVALUATION GUIDELINES - Course paper

EBA 29103
Mathematics for Business Analytics

Department of Economics

Start date:	15.03.2021	Time 09:00
Finish date:	22.03.2021	Time 12:00

For more information about formalities, see examination paper.

Solutions	EBA 29103 Mathematics for Business Analytics
Date	March 22th 2021 at 1200

Each sub-question has maximal score 6p, and the exam has maximal score 144p. To pass the exam, a score of approximately 86p is required. You may self-assess and use the table below to enter your scores. When we evaluate your answers, we emphasize the choice of method (you should give reasons based on theory where it is necessary), and the execution (that the computations are correct). To obtain the correct answer is less important, and there are in most cases alternative ways of writing the answer that will give full score.

Question	1.	2.	3.	4.	5.	6.	7.	8.	9.	Total	Grade	A	B	C	D	E
Points	24	12	24	18	12	6	18	18	12	144	Limits	130	110	84	66	58
Score																

Question 1. 24 P.

a) $\int 10x\sqrt{x}\;\mathrm{d}x = \int 10x^{3/2}\;\mathrm{d}x = 10(2/5)x^{5/2} + C = 4x^2\sqrt{x} + C$ 6 P.
b) $\int \frac{2x-1}{x^2}\;\mathrm{d}x = \int 2/x - x^{-2}\;\mathrm{d}x = 2\ln|x| + x^{-1} = 2\ln|x| + 1/x + C$ 6 P.
c) $\int 4x(1-x^2)\;\mathrm{d}x = \int 4x - 4x^3\;\mathrm{d}x = 2x^2 - x^4 + C$ 6 P.
d) $\int 12(1+4x)^2\;\mathrm{d}x = \int 12u^2 \cdot 1/4\;\mathrm{d}u = u^3 + C = (1+4x)^3 + C$ 6 P.

Question 2. 12 P.

(a) We write down the aumented matrix of the system, mark the first pivot position, and make elementary row operations using the first pivot to eliminate the entries below it:

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 5 & 8 & -2 & 23 \\ 2 & 6 & -5 & 6 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -2 & 3 & 8 \\ 2 & 6 & -5 & 6 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -2 & 3 & 8 \\ 0 & 2 & -3 & 0 \end{array}\right)$$

We mark the pivot position in the second row, and use it to eliminate the number below it:

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -2 & 3 & 8 \\ 0 & 2 & -3 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -2 & 3 & 8 \\ 0 & 0 & 0 & 8 \end{array}\right) \; 3\,\mathrm{P.}$$

We obtain an echelon form, and we see that there is an echelon in the last column. Hence there are no solutions. 3 P.

(b) We write down the aumented matrix of the system, mark the first pivot position, and make elementary row operations using the first pivot to eliminate the entries below it:

$$\left(\begin{array}{cccc|c} 1 & 2 & 4 & 1 & 11 \\ 4 & 9 & 12 & -1 & 40 \\ 5 & 10 & 16 & 1 & 51 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 2 & 4 & 1 & 11 \\ 0 & 1 & -4 & -5 & -4 \\ 5 & 10 & 16 & 1 & 51 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 2 & 4 & 1 & 11 \\ 0 & 1 & -4 & -5 & -4 \\ 0 & 0 & -4 & -4 & -4 \end{array}\right)$$

We mark the pivot position in the second row and third row, and notice that we have an echelon form:

$$\left(\begin{array}{cccc|c} 1 & 2 & 4 & 1 & 11 \\ 0 & 1 & -4 & -5 & -4 \\ 0 & 0 & -4 & -4 & -4 \end{array}\right) \; 3\,\mathrm{P.}$$

Hence there are infinitely many solutions, with w free since there is no pivot in the w-column. We find them by back substitution:

$$\begin{aligned} -4z - 4w = -4 \quad &\Rightarrow \quad -4z = -4 + 4w & z &= 1 - w \\ y - 4z - 5w = -4 \quad &\Rightarrow \quad y = 4(1-w) + 5w - 4 & y &= w \\ x + 2y + 4z + w = 11 \quad &\Rightarrow \quad x = 11 - 2(w) - 4(1-w) - w & x &= 7 + w \end{aligned}$$

The solutions are $(x,y,z,w) = (7+w, w, 1-w, w)$ where w is a free variable. 3 P.

Question 3. 24 P.

(a) We use the substitution $u = 1 + e^x$, which gives $\mathrm{d}u = u' \,\mathrm{d}x = e^x \,\mathrm{d}x$. 3 P. This gives

$$\int \frac{e^x}{1+e^x}\,\mathrm{d}x = \int \frac{e^x}{u}\frac{\mathrm{d}u}{e^x} = \int \frac{1}{u}\,\mathrm{d}u = \ln|u| + C = \ln(1+e^x) + C \text{ 3 P.}$$

(b) We factorize the denominator as $1 - 4x^2 = (1+2x)(1-2x)$, and simplify the expression using partial fractions. This gives

$$\frac{1-x}{1-4x^2} = \frac{A}{1+2x} + \frac{B}{1-2x} \quad \Rightarrow \quad 1 - x = A(1-2x) + B(1+2x)$$

Hence $1 - x = (A+B) + (2B - 2A)x$, or $A + B = 1$ and $2B - 2A = -1$. This linear system gives $B = 1/4$ and $A = 3/4$, 3 P. and the integral becomes

$$\int \frac{1-x}{1-4x^2}\,\mathrm{d}x = \int \frac{3}{4}\frac{1}{1+2x} + \frac{1}{4}\frac{1}{1-2x}\,\mathrm{d}x = \frac{3}{4}\frac{1}{2}\ln|1+2x| + \frac{1}{4}\frac{1}{(-2)}\ln|1-2x| + C$$
$$= \frac{3}{8}\ln|1+2x| - \frac{1}{8}\ln|1-2x| + C \text{ 3 P.}$$

(c) We use the substitution $u = \ln x$, which gives $\mathrm{d}u = (1/x)\,\mathrm{d}x$, 3 P. and this gives

$$\int \frac{3(\ln x)^2}{x}\,\mathrm{d}x = \int \frac{3u^2}{x}\cdot\frac{\mathrm{d}u}{1/x} = \int 3u^2\,\mathrm{d}u = u^3 + C = (\ln x)^3 + C \text{ 3 P.}$$

Alternatively, we may use integration by parts with $u' = 1/x$ and $v = (\ln x)^2$.

(d) We use the substitution $u = -x\sqrt{x} = -x^{3/2}$, which gives $\mathrm{d}u = u'\,\mathrm{d}x = (-3/2)x^{1/2}\,\mathrm{d}x$. This gives

$$\int 6x^2\,e^{-x\sqrt{x}}\,\mathrm{d}x = \int 6x^2\,e^u\,\frac{\mathrm{d}u}{(-3/2)x^{1/2}} = \int -4x^{3/2}\,e^u\,\mathrm{d}u = \int 4ue^u\,\mathrm{d}u \text{ 3 P.}$$
$$= 4ue^u - \int 4e^u\,\mathrm{d}u = 4ue^u - 4e^u + C = (-4x\sqrt{x} - 4)e^{-x\sqrt{x}} + C \text{ 3 P.}$$

To go from the first to the second line, we use integration by parts.

Question 4. 18 P.

(a) The determinant is given by

$$|A| = \begin{vmatrix} 6 & 2a \\ a & 3 \end{vmatrix} = 18 - 2a^2 = 2(9 - a^2) = 2(3-a)(3+a) \text{ 3 P.}$$

Hence $|A| = 0$ when $a = \pm 3$. 3 P.

(b) We compute the determinant of A using cofactor expansion along the first row:

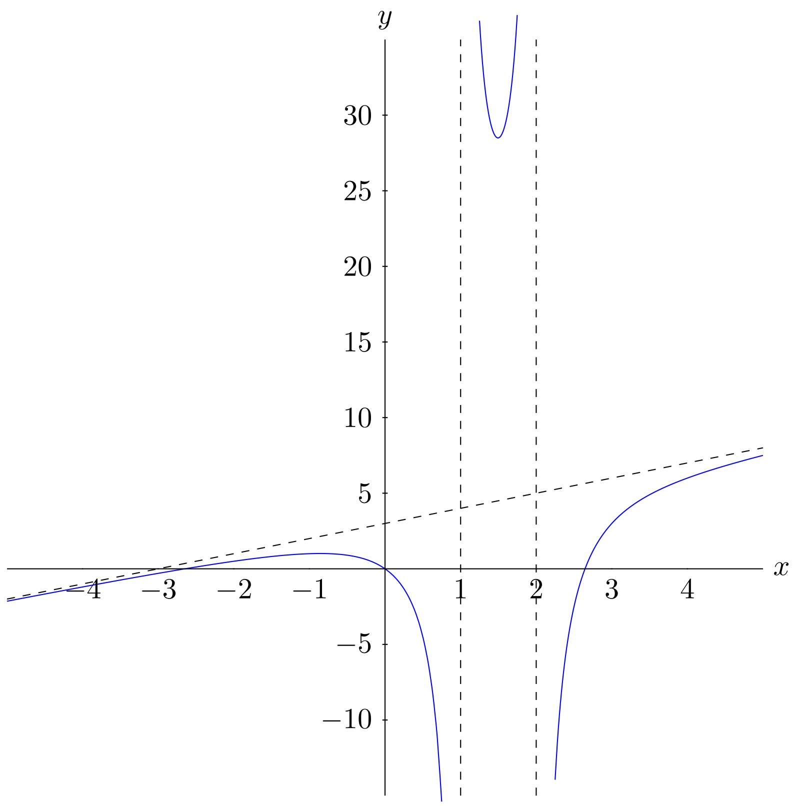
$$|A| = \begin{vmatrix} 1 & 1 & s \\ 1 & 2 & s \\ s & 3 & 9 \end{vmatrix} = 1(18 - 3s) - 1(9 - s^2) + s(3 - 2s) = -s^2 + 9 = (3-s)(3+s) \text{ 3 P.}$$

Hence $|A| = 0$ when $s = \pm 3$. 3 P.

(c) We compute the determinant of A using cofactor expansion along the first row:

$$|A| = \begin{vmatrix} t & 1 & 4 \\ 1 & t & 4 \\ 1 & 4 & t \end{vmatrix} = t(t^2 - 16) - 1(t - 4) + 4(4 - t) = (t-4)\left[t(t+4) - 1 - 4\right]$$
$$= (t-4)(t^2 + 4t - 5) = (t-4)(t-1)(t+5) \text{ 3 P.}$$

We have factored out $(t-4)$, which is a common factor in all three terms. Hence $|A| = 0$ when $t = 1, t = 4, t = -5$. 3 P.



Question 5. 12 P.

(a) Since $x^2 - 3x + 2 = (x-1)(x-2)$, we have vertical asymptotes $x = 1$ and $x = 2$. 1 P. Polynomial division gives $f(x) = x + 3 - 6/(x^2 - 3x + 2)$, hence f has a skew asymptote L with equation $y = x + 3$. 2 P. The figure is shown above. 3 P.

(b) We see that the zeros of f are given by $x^3 - 7x = 0$, or $x = 0$ and $x = \pm\sqrt{7}$. The region R in the second quadrant bounded by the graph of f and the x-axis can be found in the interval $-\sqrt{7} \le x \le 0$, and it has area given by

$$\begin{aligned} A = \int_{-\sqrt{7}}^{0} f(x)\ \mathrm{d}x &= \int_{-\sqrt{7}}^{0} x + 3 - \frac{6}{(x-1)(x-2)}\ \mathrm{d}x \\ &= \left[\frac{1}{2}x^2 + 3x\right]_{-\sqrt{7}}^{0} - \int_{-\sqrt{7}}^{0} \frac{6}{(x-1)(x-2)}\ \mathrm{d}x \quad 3\,\text{P.} \end{aligned}$$

The first term is given by

$$\left[\frac{1}{2}x^2 + 3x\right]_{-\sqrt{7}}^{0} = 0 - (7/2 - 3\sqrt{7}) = 3\sqrt{7} - 7/2$$

To compute the last integral, we use partial fractions:

$$\frac{6}{x^2 - 3x + 2} = \frac{A}{x-1} + \frac{B}{x-2} \quad \Rightarrow \quad 6 = A(x-2) + B(x-1)$$

We find the constants A and B by substituting $x = 1$ and $x = 2$, and this gives $A = -6$ and $B = 6$. We get

$$\int_{-\sqrt{7}}^{0} \frac{6}{(x-1)(x-2)}\ \mathrm{d}x = \int_{-\sqrt{7}}^{0} \frac{-6}{x-1} + \frac{6}{x-2}\ \mathrm{d}x = \left[-6\ln|x-1| + 6\ln|x-2|\ \right]_{-\sqrt{7}}^{0}$$

which gives

$$\left[6\ln\frac{|x-2|}{|x-1|}\right]_{-\sqrt{7}}^{0} = 6\ln(2) - 6\ln\frac{2+\sqrt{7}}{1+\sqrt{7}}$$

Therefore the area A of the region R is given by

$$A = 3\sqrt{7} - 7/2 - 6\ln(2) + 6\ln\frac{2+\sqrt{7}}{1+\sqrt{7}} \approx 1.73 \quad 3\,\text{P.}$$

Question 6. 6 P.

We first compute the determinant of the coefficient matrix A of the linear system by cofactor expansion along the first row:

$$|A| = \begin{vmatrix} 1 & 2 & a \\ a & 3 & 5 \\ a & 0 & 1 \end{vmatrix} = a(10-3a) + 1(3-2a) = -3a^2 + 8a + 3 = (3-a)(1+3a) \text{ 1 P.}$$

We see that $a = 3$ and $a = -1/3$ gives $|A| = 0$. This means that the system has no solutions or infinitely many solutions for these three values of a, and exactly one solution otherwise. 2 P. We first consider the case $a = 3$, and solve the system by Gaussian elimination:

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 3 & 3 & 5 & 3 \\ 3 & 0 & 1 & 3 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -3 & -4 & 0 \\ 0 & -6 & -8 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -3 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

We see that the system has infinitely many solutions for $a = 3$, with z as a free variable, and the solutions are given by

$$-3y - 4z = 0 \quad \Rightarrow \quad y = -4z/3 \quad \text{og} \quad x + 2y + 3z = 1 \quad \Rightarrow \quad x = 1 - 3z - 2(-4z/3) = 1 - z/3$$

Hence the solutions are $(x,y,z) = (1 - z/3, -4z/3, z)$ med z fri when $a = 3$. 1 P. When $a = -1/3$, we solve the system by Gaussian elimination. We first multiply all rows with 3 and switch the first and last row:

$$\left(\begin{array}{ccc|c} 1 & 2 & -1/3 & 1 \\ -1/3 & 3 & 5 & -1/3 \\ -1/3 & 0 & 1 & 3 \end{array}\right) \to \left(\begin{array}{ccc|c} 3 & 6 & -1 & 3 \\ -1 & 9 & 15 & -1 \\ -1 & 0 & 3 & 9 \end{array}\right) \to \left(\begin{array}{ccc|c} -1 & 0 & 3 & 9 \\ -1 & 9 & 15 & -1 \\ 3 & 6 & -1 & 3 \end{array}\right)$$

Then we find an echelon form:

$$\left(\begin{array}{ccc|c} -1 & 0 & 3 & 9 \\ -1 & 9 & 15 & -1 \\ 3 & 6 & -1 & 3 \end{array}\right) \to \left(\begin{array}{ccc|c} -1 & 0 & 3 & 9 \\ 0 & 9 & 12 & -10 \\ 0 & 6 & 8 & 30 \end{array}\right) \to \left(\begin{array}{ccc|c} -1 & 0 & 3 & -9 \\ 0 & 9 & 12 & 8 \\ 0 & 0 & 0 & 110/3 \end{array}\right)$$

Hence, the system has no solutions when $a = -1/3$. 1 P. When $a \neq 3$ and $a \neq -1/3$, there are exactly one solutions, and we find it using Kramer's rule:

$$|A_1(\mathbf{b})| = \begin{vmatrix} 1 & 2 & a \\ a & 3 & 5 \\ 3 & 0 & 1 \end{vmatrix} = 33 - 11a \quad \Rightarrow \quad x = \frac{11(3-a)}{(3-a)(1+3a)} = \frac{11}{1+3a}$$

$$|A_2(\mathbf{b})| = \begin{vmatrix} 1 & 1 & a \\ a & a & 5 \\ a & 3 & 1 \end{vmatrix} = -a^3 + 3a^2 + 5a - 15 \quad \Rightarrow \quad y = \frac{(3-a)^(a^2-5)}{(3-a)(1+3a)} = \frac{a^2-5}{1+3a}$$

$$|A_3(\mathbf{b})| = \begin{vmatrix} 1 & 2 & 1 \\ a & 3 & a \\ a & 0 & 3 \end{vmatrix} = 2a^2 - 9a + 9 \quad \Rightarrow \quad z = \frac{(3-a)(3-2a)}{(3-a)(1+3a)} = \frac{3-2a}{1+3a} \text{ 1 P.}$$

Question 7. 18 P.

(a) We have that

$$A^2 = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} \text{ 6 P.}$$

(b) We compute the determinant $|A| = 1(-1) - 1(1) = -2$ using cofactor expansion along the first row. It follows that

$$A^{-1} = \frac{1}{-2} \begin{pmatrix} -1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & -1 \end{pmatrix}^T = \frac{1}{2} \begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{pmatrix} \text{ 6 P.}$$

(c) We have that

$$AB+BA=\begin{pmatrix}1&1&0\\1&0&1\\0&1&1\end{pmatrix}\cdot\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}+\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}\cdot\begin{pmatrix}1&1&0\\1&0&1\\0&1&1\end{pmatrix}$$
$$=\begin{pmatrix}0&1&1\\1&0&1\\1&1&0\end{pmatrix}+\begin{pmatrix}0&1&1\\1&0&1\\1&1&0\end{pmatrix}=\begin{pmatrix}0&2&2\\2&0&2\\2&2&0\end{pmatrix}\ 6\,\text{P.}$$

Question 8. 18 P.

(a) The present value of the cash flow from renting the property is

$$\int_0^\infty I(t)e^{-rt}\,\mathrm{d}t=\int_0^\infty 10\,e^{0.06t}\,e^{-0.10t}\,\mathrm{d}t=\int_0^\infty 10\,e^{-0.04t}\,\mathrm{d}t\ 3\,\text{P.}$$
$$=\left[\frac{10}{-0.04}\,e^{-0.04t}\right]_0^\infty=\lim_{b\to\infty}\left[-250\,e^{-0.04t}\right]_0^b=\lim_{b\to\infty}-250(e^{-0.04b}-1)=250\ 3\,\text{P.}$$

(b) Let $S(t)$ be the present value of the sale price of the property when we sell it after t years. Then we have that

$$S(t)=V(t)e^{-rt}=250\,e^{\sqrt{t}/5}\,e^{-0.10t}=250\,e^{(2\sqrt{t}-t)/10}\ 1\,\text{P.}$$

To maximize $S(t)$, we find the derivative. We use $u=(2\sqrt{t}-t)/10$ as kernel, and find that

$$S'(t)=250\,e^u\cdot u'=250\,e^u\cdot\frac{1}{10}\left(\frac{2}{2\sqrt{t}}-1\right)=25\,e^u\cdot\frac{1-\sqrt{t}}{\sqrt{t}}\ 3\,\text{P.}$$

Hence $S'(t)=0$ when $1-\sqrt{t}=0$, and this gives $\sqrt{t}=1$, or $t=1$. The remaining factors in the expression for $S'(t)$ are positive, and $1-\sqrt{t}$ is changing sign from positive to negativ at $t=1$. This means that $t=1$ is a maximum point for the function $S(t)$. The present value of the sale sum is maximal after one year. 2 P.

(c) We have that

$$N(T)=\int_0^T I(t)e^{-rt}\,\mathrm{d}t+V(T)e^{-rT}\ 1\,\text{P.}$$

The first term is given by

$$\int_0^T I(t)e^{-rt}\,\mathrm{d}t=\int_0^T 10\,e^{0.06t}\,e^{-0.10t}\,\mathrm{d}t=\int_0^T 10e^{-0.04t}\,\mathrm{d}t=\left[\frac{10}{-0.04}\,e^{-0.04t}\right]_0^T$$
$$=-250(e^{-0.04T}-1)=250(1-e^{-0.04T})$$

and the second term is

$$S(T)=250\,e^{(2\sqrt{T}-T)/10}$$

Hence the total present value is given by

$$N(T)=250(1-e^{-0.04T})+250\,e^{(2\sqrt{T}-T)/10}=250\left(1-e^{-0.04T}+e^{(2\sqrt{T}-T)/10}\right)\ 3\,\text{P.}$$

After 0, 1, 2 and 3 year, the total present value is:

a) $N(0)=250(1-1+1)=250$

b) $N(1)=250(1-e^{-0.04}+e^{0.10})\approx 286.1$

c) $N(2)=250(1-e^{-0.08}+e^{(\sqrt{2}-1)/5})\approx 290.8$

d) $N(3)=250(1-e^{-0.12}+e^{(2\sqrt{3}-3)/10})\approx 290.1$ 1 P.

It seems that the total present value is maximal after between 2 and 3 years. 1 P.

	Return A	Return B	Return C
Scenario 1	-25	25	-150
Scenario 2	50	-75	120
Scenario 3	20	15	20

Question 9. 12 P.

We find the return per share for the different companies and the different scenarios, see the table above. We use this to express the total return R_i using x,y,z, which gives three linear equations. In addition, we have the budget condition $100x + 125y + 284z = C$, which specify that the total cost is $C = 1.500.000$. We find the following linear system and extended matrix:

$$\begin{array}{rcrcrcl} -25x &+& 25y &-& 150z &=& R_1 \\ 50x &-& 75y &+& 120z &=& R_2 \\ 20x &+& 15y &+& 20z &=& R_3 \\ 100x &+& 125y &+& 284z &=& C \end{array} \quad \Rightarrow \quad \left(\begin{array}{rrr|l} -25 & 25 & -150 & R_1 \\ 50 & -75 & 120 & R_2 \\ 20 & 15 & 20 & R_3 \\ 100 & 125 & 284 & C \end{array}\right)$$

(a) We first solve the system when $(R_1, R_2, R_3) = (1.000.000, -2.000.000, 200.000)$. This gives the following echelon form:

$$\left(\begin{array}{rrr|l} -25 & 25 & -150 & R_1 \\ 50 & -75 & 120 & R_2 \\ 20 & 15 & 20 & R_3 \\ 100 & 125 & 284 & C \end{array}\right) \to \left(\begin{array}{rrr|l} -25 & 25 & -150 & R_1 \\ 0 & -25 & -180 & R_2 + 2R_1 \\ 0 & 35 & -100 & R_3 + 0.8R_1 \\ 0 & 225 & -316 & C + 4R_1 \end{array}\right)$$

$$\to \left(\begin{array}{rrr|l} -25 & 25 & -150 & R_1 \\ 0 & -25 & -180 & R_2 + 2R_1 \\ 0 & 0 & -352 & R_3 + 1.4R_2 + 3.6R_1 \\ 0 & 0 & -1936 & C + 9R_2 + 22R_1 \end{array}\right)$$

$$\to \left(\begin{array}{rrr|l} -25 & 25 & -150 & R_1 \\ 0 & -25 & -180 & R_2 + 2R_1 \\ 0 & 0 & -352 & R_3 + 1.4R_2 + 3.6R_1 \\ 0 & 0 & 0 & C - 5.5R_3 + 1.3R_2 + 2.2R_1 \end{array}\right) \quad 3\,\text{P.}$$

Since $C = 1.500.000$ and $(R_1, R_2, R_3) = (1.000.000, -2.000.000, 200.000)$, the expression $C + 2.2R_1 + 1.3R_2 - 5.5R_3 = 0$. This means that the system has a unique solution, and there is a portfolio with the specified returns. 1 P. We find this portfolio by back substitution:

$$\begin{array}{rcrcrcr} -25x &+& 25y &-& 150z &=& 1.000.000 \\ &-& 25y &-& 180z &=& 0 \\ &&&-& 352z &=& 1.000.000 \end{array}$$

which gives

$$(x,y,z) = (-2.500, 20.454\,{}^{6}\!/\!_{11}, -2.840\,{}^{10}\!/\!_{11}) \ 2\,\text{P.}$$

(b) To find all possible returns (R_1, R_2, R_3), we use the same linear system as above, and obtain the same echelon form. it follows that the possible returns are the triples (R_1, R_2, R_3) that satisfy

$$C + 2.2R_1 + 1.3R_2 - 5.5R_3 = 0 \quad \Rightarrow \quad -2.2R_1 - 1.3R_2 + 5.5R_3 = 1.500.000 \ 2\,\text{P.}$$

There are many solutions with $R_1, R_2, R_3 > 0$. We choose $R_1 = R_2 = R_3$ which gives

$$2R_1 = 1.500.000 \quad \Rightarrow \quad R_1 = 750.000$$

This choice gives $R_1 = R_2 = R_3 = 750.000$. 2 P. To find the portfolio with these returns, we solve

$$\begin{array}{rcrcrcr} -25x &+& 25y &-& 150z &=& 750.000 \\ &-& 25y &-& 180z &=& 2.250.000 \\ &&&-& 352z &=& 4.500.000 \end{array}$$

by back substitution. This gives

$$(x,y,z) = (48.750, 2.045\,{}^{5}\!/\!_{11}, -12.784\,{}^{1}\!/\!_{11}) \ 2\,\text{P.}$$