EXAMINATION QUESTION PAPER - Course paper

EBA 29103
Mathematics for Business Analytics

Department of Economics

Start date:	15.03.2021	Time 09.00
Finish date:	22.03.2021	Time 12.00

Weight:	Pass / Fail
Total no. of pages:	3 incl. front page
No. of attachments files to question paper:	0
To be answered:	Individually

Answer paper size:	No limit. excl. attachments
Max no. of answer paper attachment files:	0
Allowed answer paper file types:	pdf

Course paper	EBA 29103 Mathematics for Business Analytics
Deadline	March 22th 2021 at 1200

The problem set consists of two pages. All 24 subquestion have equal weight, and at least 60% score is required to pass. **You must give reasons for your answers. Precision and clarity will be emphasized when evaluating your answers.**

Your answers should be provided as a single file in PDF format. You are encouraged to write with a pen and scan your paper. Check that the resulting file is easy to read. For more information, see `https://portal.bi.no/en/examination/digital-examination/digital-submission/`.

Question 1.
Compute the indefinite integrals:

a) $\int 10x\sqrt{x}\,\mathrm{d}x$ b) $\int \frac{2x-1}{x^2}\,\mathrm{d}x$ c) $\int 4x(1-x^2)\,\mathrm{d}x$ d) $\int 12(1+4x)^2\,\mathrm{d}x$

Question 2.
Use Gaussian elimination to solve the linear systems. Show elementary row operations, mark the pivot positions in the echelon form, and specify the number of solutions.

$$a)\quad \begin{aligned} x + 2y - z &= 3 \\ 5x + 8y - 2z &= 23 \\ 2x + 6y - 5z &= 6 \end{aligned} \qquad b)\quad \begin{aligned} x + 2y + 4z + w &= 11 \\ 4x + 9y + 12z - w &= 40 \\ 5x + 10y + 16z + w &= 51 \end{aligned}$$

Question 3.
Compute the integrals:

a) $\int \frac{e^x}{1+e^x}\,\mathrm{d}x$ b) $\int \frac{1-x}{1-4x^2}\,\mathrm{d}x$ c) $\int \frac{3(\ln x)^2}{x}\,\mathrm{d}x$ d) $\int 6x^2\,e^{-x\sqrt{x}}\,\mathrm{d}x$

Question 4.
Compute the determinant $|A|$, and determine when $|A| = 0$:

a) $A = \begin{pmatrix} 6 & 2a \\ a & 3 \end{pmatrix}$ b) $A = \begin{pmatrix} 1 & 1 & s \\ 1 & 2 & s \\ s & 3 & 9 \end{pmatrix}$ c) $A = \begin{pmatrix} t & 1 & 4 \\ 1 & t & 4 \\ 1 & 4 & t \end{pmatrix}$

Question 5.

We consider the function $f(x) = \dfrac{x^3 - 7x}{x^2 - 3x + 2}$.

(a) Determine the asymptotes of the function f, and make a figure.
(b) We let R be the area in the second quadrant bounded by the graph of f and the x-axis. Mark the area R in the figure, and compute the area of R.

Question 6.
Determine how many solutions the linear system has for different values of the parameter a, and find all solutions in the cases when the system is consistent:

$$\begin{aligned} x + 2y + az &= 1 \\ ax + 3y + 5z &= a \\ ax + z &= 3 \end{aligned}$$

Question 7.
Compute the following expressions when the matrices A and B are given by

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

a) A^2 b) A^{-1} c) $AB + BA$

Question 8.
We purchase a property for 250 MNOK (million kroner). We assume that the value $V(t)$ of the property (in MNOK) is given by

$$V(t) = 250\, e^{\sqrt{t}/5}$$

after t years. If we choose to rent out the property, we expect to receive a rent $I(t)$ (in MNOK per year). We consider this as a continuous cash flow, and assume that it is given by

$$I(t) = 10\, e^{0.06t}$$

after t year. We use continuous discounting when we compute present values, with discount rate $r = 10\%$.

(a) Find the present value of the cash flow we would received for renting the property with an infinite time horizon.
(b) We consider selling the property at some time in the future. When is the present value of the property value maximal?
(c) Assume that we rent out the property for T years, and then sell the property. Write down an expression for the total present value $N(T)$ of the cash flow for renting out and selling the property, and compute $N(T)$ when
(i) $T = 0$ (ii) $T = 1$ (iii) $T = 2$ (iv) $T = 3$
At what time does it seem optimal to sell the property?

Question 9.
You have 1.500.000 kr that you will invest in a portfolio of shares. You can choose a combination of the companies A, B, C with price $p_A = 100$ kr, $p_B = 125$ kr and $p_C = 284$ kr per share at the time when you invest. We assume that at a given time in the future, one of three scenarios will happen, and the prices for the shares in these scenarios are given in the table below. We write x, y, z for

	Price A	Price B	Price C
Cost	100	125	284
Scenario 1	75	150	134
Scenario 2	150	50	404
Scenario 3	120	140	304

the number of shares you buy in each of the three companies, and we assume that x,y,z can be any real numbers. This means that we allow buying a negative number of shares (short selling), and the number of shares does not need to be an integer.

(a) We write R_1, R_2 og R_3 for the return of the portfolio in the three scenarios. Is it possible to choose a portfolio with $(R_1, R_2, R_3) = (1.000.000, -2.000.000, 200.000)$? If so, how many shares do we need to buy of each of the three companies?
(b) Determine all triples (R_1,R_2,R_3) of possible returns in the three scenarios. Are there any portfolios with $R_1, R_2, R_3 > 0$ (positive return in any scenario)? If so, specify one such portfolio (the number of shares in each company, and the corresponding returns).