Course paper	EBA 29103 Mathematics for Business Analytics
Deadline	March 21st 2022 at 1200

The problem set consists of two pages. All 24 subquestion have equal weight, and at least 60% score is required to pass. **You must give reasons for your answers. Precision and clarity will be emphasized when evaluating your answers.** Your answers should be provided as a single file in PDF format.

Question 1.
Compute the integrals:

a) $\displaystyle\int_0^1 12x^2 + 3\sqrt{x}\,\mathrm{d}x$ b) $\displaystyle\int_1^2 9\sqrt{x}\ln x\,\mathrm{d}x$ c) $\displaystyle\int_3^\infty \frac{6}{x^2-1}\,\mathrm{d}x$ d) $\displaystyle\int_0^\infty \frac{1}{\sqrt{x}+1}\,\mathrm{d}x$

Question 2.
Compute the integrals:

a) $\displaystyle\int \frac{x+9}{x^2-3x-10}\,\mathrm{d}x$ b) $\displaystyle\int \frac{\ln(x-1)}{x^2}\,\mathrm{d}x$ c) $\displaystyle\int \frac{1}{e^x+1}\,\mathrm{d}x$

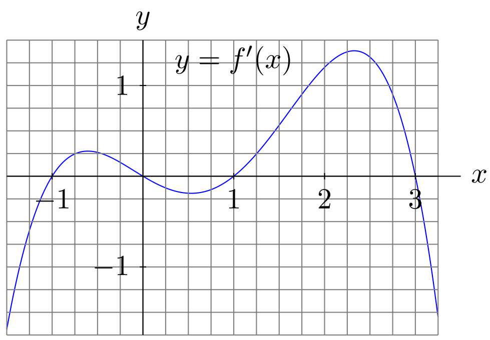
Question 3.
Let E be the ellipse with symmetry lines $x = 2$ and $y = 1$ going through the points (5,1) and (2,3), and let H be the hyperbola going through the point (2,3) with $x = -1$ and $y = -1$ as asymptotes.

a) Find the equation of the ellipse E and the hyperbola H.
b) Make a figure showing E, H, and the area R in the first quadrant bounded by E and H, and compute the area of R. You may use that the area of an ellipse with half-axes a,$b > 0$ is given by πab.

Question 4.
Let $f(x)$ be the net cash flow after x years (in million NOK per year) from a rental property. We think of this as a continuous cash flow, and use continuous discounting with discount rate $r = 10\%$ to compute net present values. Find the total net present value from the rental property in the first 10 years when

a) $f(x) = 100 + 4x$ b) $f(x) = 100 \cdot 1.04^x$

Question 5.
The graph of $f'(x)$ is shown in the figure below. Use the figure to find the global maximum point $x = a$ of the function f, if it exists. Estimate the value of the integral $\int_0^3 f'(x)\,\mathrm{d}x$.



Question 6.
Use Gaussian elimination to solve the linear systems. Show the elementary row operations, mark the pivot positions in the echelon form, and specify the number of solutions:

$$a)\quad \begin{array}{rcrcrcr} x & + & 2y & - & z & = & 3 \\ 5x & + & 8y & - & 2z & = & 23 \\ 2x & + & 6y & - & 5z & = & 6 \\ 4x & + & 4y & + & 2z & = & 21 \end{array} \qquad\qquad b)\quad \begin{array}{rcrcrcrcr} x & + & 2y & + & 4z & + & w & = & 11 \\ 2x & + & 5y & + & 4z & - & 3w & = & 18 \\ 4x & + & 8y & + & 12z & & & = & 28 \end{array}$$

Question 7.
Compute the determinant $|A|$, and determine when $|A| = 0$:

$$\text{a) } A = \begin{pmatrix} 2 & 1 & 4 \\ 3 & 2 & 1 \\ 1 & a & 7 \end{pmatrix} \qquad \text{b) } A = \begin{pmatrix} 0 & s & 1 \\ s & 0 & 1 \\ 1 & 1 & s \end{pmatrix} \qquad \text{c) } A = \begin{pmatrix} 1 & t & 0 & 0 \\ t & 1 & 0 & 0 \\ 0 & 0 & t & 8 \\ 0 & 0 & 2 & t \end{pmatrix}$$

Question 8.
Let $\mathbf{v}_1 = (1{,}3{,}2{,}4)$, $\mathbf{v}_2 = (2{,}5{,}6{,}7)$, og $\mathbf{v}_3 = (5{,}11{,}4{,}9)$.

a) Determine whether the vector $\mathbf{w} = (3{,}4{,}6{,}2)$ is a linear combination of the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

b) Determine all vectors that are linear combinations of the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

Question 9.
The linear system $A\mathbf{x} = \mathbf{b}$ is given by

$$A = \begin{pmatrix} 3 & 7 & a \\ 2 & 5 & 3 \\ 5 & a & 35 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -8 \\ 4 \\ -144 \end{pmatrix}$$

where a is a parameter.

a) Find A^{-1} when $a = 0$.

b) Determine all values of a such that $A\mathbf{x} = \mathbf{b}$ has a unique solution.

c) Find all solutions of $A\mathbf{x} = \mathbf{b}$ in the cases where there are infinitely many solutions.

d) Find the z-coordinate of the solution (x,y,z) in the cases where $A\mathbf{x} = \mathbf{b}$ has a unique solution.

Question 10.
Solve the matrix equation for X:

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} X \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & 2 \end{pmatrix}$$