EXAMINATION QUESTION PAPER - Take-home examination

EBA 09101
Mathematics for Business Analytics

Department of Economics

Start date:	19.06.2020	Time 09.00
Finish date:	19.06.2020	Time 16.00

Weight:	100% of EBA 0910
Total no. of pages:	4 incl. front page
No. of attachments files to question paper:	0
To be answered:	Individually

Answer paper size:	No limit. excl. attachments
Max no. of answer paper attachment files:	0
Allowed answer paper file types:	pdf

Exam	EBA2910 Mathematics for Business Analytics
Date	June 19th 2020 at 0900 - 1600

The following applies to this exam:

a) The answer paper must be written and prepared individually. Collaboration with others is not permitted and is considered cheating.

b) All answer papers are automatically subjected to plagiarism control. Students may also be called in for an oral consultation as additional verification of an answer paper.

c) Your answers must be written by hand. All answers must be justified based on the theory in the course, and the justifications are especially important for home exams.

Please add your ID number (7 digits), top right, on all pages of your answer paper.

Question 1.

The functions f and g have domain of definition $[-1,3]$, and the graphs of the derivatives $f'(x)$ and $g'(x)$ are shown in Figure 1.

a) Find the slopes of the tangents of the functions f and g at $x = 1$.

b) Find the x-coordinates of the minimum points of the functions f and g.

c) Determine whether the functions f and g have inverse functions.

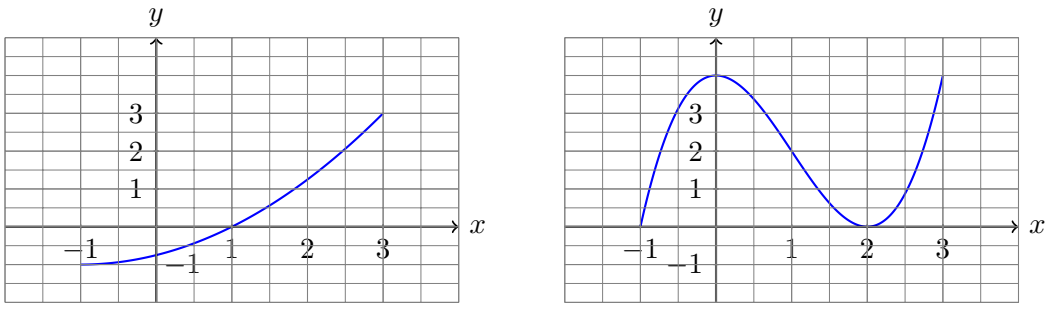

FIGURE 1. The graph of $y = f'(x)$ on the left and of $y = g'(x)$ on the right

Question 2.

Consider the linear system $A \cdot \mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} 3 & a & -2 \\ a & a^2+1 & -a \\ -2 & -a & 3 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

and a is a parameter.

a) Solve the linear system using Gaussian elimination when $a = 2$. Show all row operations.

b) Compute $\det(A)$, and determine all values of a a such that $A\,\mathbf{x} = \mathbf{b}$ has exactly one solution.

c) Find A^{-1} when $a = 0$.

d) Determine the value of $A^n \cdot \mathbf{b}$ when n is a large integer and $a = 2$.

Question 3.

The rational function f is given by the functional expression $f(x) = Q(x)/L(x)$, where $Q(x)$ is a quadratic polynomial and $L(x)$ is a linear polynomial. The graph of f is shown in Figure 2.

a) Find the asymptotes of f.

b) Find the functional expression of f, and compute $f'(x)$.

c) Determine whether f has global maxima or minima.

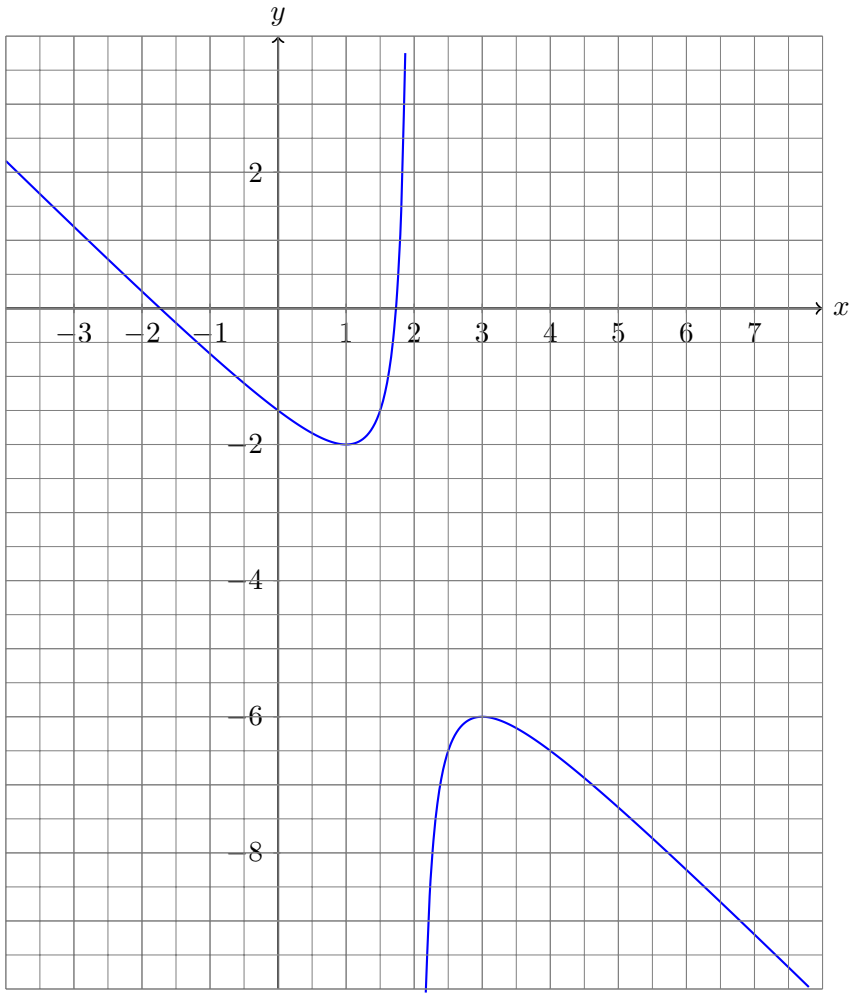

FIGURE 2. The graph of $y = f(x)$

Question 4.
Compute the integrals. Show the integration rules you use.

a) $\int x(1-x)^2 \, \mathrm{d}x$ b) $\int \frac{x}{1-x^2} \, \mathrm{d}x$ c) $\int \frac{x}{(1-\sqrt{x})^2} \, \mathrm{d}x$

The function f is defined for $-4 \leq x \leq 3$ and has the graph shown in Figure 3.

d) Which value of a gives the greatest value for the definite integral $\int_{-4}^{a} f(x) \, \mathrm{d}x$?

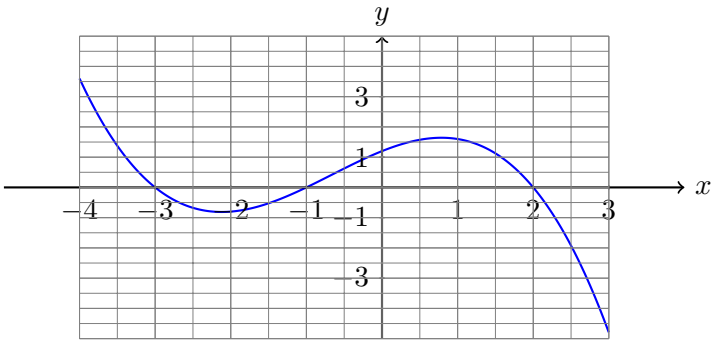

FIGURE 3. The graph of $y = f(x)$

Question 5.
Consider the function $f(x) = xe^x$.

a) Explain why the equation $f(x) = -1$ does not have any solutions.

b) Let $W = f^{-1}$ be the inverse function of $f(x) = xe^x$, $x \geq -1$. Determine whether W is an increasing or a decreasing function.

Question 6.
The curve C in the xy-plane is given by the equation $4x^2 - 24x + t^2y^2 = 64$, where t is a parameter.

a) Show that C is an ellipse for $t \neq 0$, and sketch the curve C for suitable values of t.

b) Find the stationary points of $f(x,y) = xy$ and classify them.

c) Solve $\max f(x,y) = xy$ when $4x^2 - 24x + 16y^2 = 64$ by Lagrange's method.

d) Show suitable level curves of $f(x,y) = xy$ in the same figure as the ellipse $4x^2 - 24x + 16y^2 = 64$, and explain the connection between these curves and the solution of the Lagrange problem.