EXAMINATION QUESTION PAPER - Take-home examination

EBA 09101
Mathematics for Business Analytics

Department of Economics

Start date:	02.06.2021	Time 10.00
Finish date:	02.06.2021	Time 15.15

Weight:	100% of EBA 0910
Total no. of pages:	3 incl. front page
No. of attachments files to question paper:	0
To be answered:	Individually

Answer paper size:	No limit. excl. attachments
Max no. of answer paper attachment files:	0
Allowed answer paper file types:	pdf
Re-sit	Ordinary

The exam paper consists of 15 questions, and 2 questions for extra credit. All answers must be justified, and the justification should be based on the theory in the course.

- **The answer paper must be handed in as a pdf file. It must be written by hand.**
- The answer paper must be written and prepared individually. Collaboration with others is not permitted and is considered cheating.
- All answer papers are automatically subjected to plagiarism control. Students may also be called in for an oral consultation as additional verification of an answer paper.

Question 1.

We consider the function $f(x) = 2\sqrt{x}\ln(x) - 4\sqrt{x}$.

(a) **(6p)** Compute $f'(x)$, and write the answer in the form $f'(x) = (c\ln(x) + d)/\sqrt{x}$.
(b) **(6p)** Determine the limits of $f(x)$ when $x \to \infty$ and when $x \to 0^+$.
(c) **(6p)** Determine the number of solutions of the equation $f(x) = a$ for all values of a.

Question 2.

Compute these indefinite integrals:

a) **(6p)** $\displaystyle\int \frac{3 - 7x}{9 - x^2}\,\mathrm{d}x$ b) **(6p)** $\displaystyle\int 15x \cdot \sqrt{x + 1}\,\mathrm{d}x$ c) **(6p)** $\displaystyle\int \frac{3\sqrt{\ln x}}{x}\,\mathrm{d}x$

Question 3.

Let f be a function such that the graph of $f'(x)$ is the hyperbola shown in Figure 1.

(a) **(6p)** Determine the asymptotes and the functional expression of the derivative f'.
(b) **(6p)** Estimate the value of $f(3) - f(2)$ using the figure.
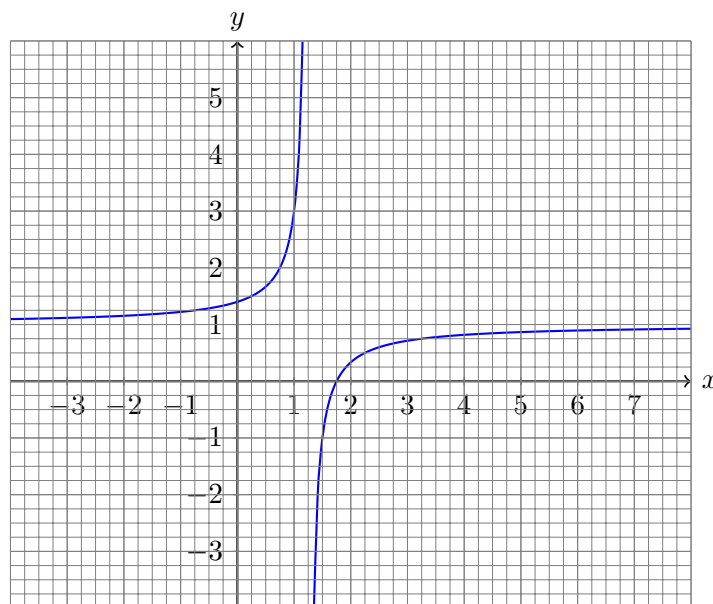(c) **(6p)** Determine $f(3) - f(2)$ using the functional expression of f'.

FIGURE 1. The graph of $f'(x)$

Question 4.

Let the matrix A and the vectors $\mathbf{x}$ and $\mathbf{b}$ be given by

$$A = \begin{pmatrix} 3 & 4 & 5 \\ 7 & 2 & 11 \\ 5 & 1 & 6 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} r \\ s \\ t \end{pmatrix}$$

- (a) **(6p)** Compute $|A|$, and determine A^{-1}.
- (b) **(6p)** Solve the linear system $A\mathbf{x} = \mathbf{b}$ when $r = 24$, $s = -20$, and $t = -6$.
- (c) **(6p)** Determine all vectors $\mathbf{b}$ such that $A\mathbf{x} = \mathbf{b}$ has solutions that satisfy $x + y + z = 9$.

Question 5.

**(6p)** Find the maximum and minimum value in the optimization problem

$$\max / \min f(x,y) = \sqrt{xy} - x \text{ when } 0 \le x \le 1, \ 0 \le y \le 1$$

Question 6.

We consider the curve C in the xy-plane given by the equation $y^2 = 5x^2 - x^3$, and the Lagrange problem

$$\max f(x,y) = x + y \text{ when } y^2 = 5x^2 - x^3$$

- (a) **(6p)** Determine all points $(x,y) \ne (0,0)$ on the curve C where the tangent line has slope -1.
- (b) **(6p)** Determine all points $(x,y; \lambda)$ that satisfy the Lagrange conditions.
- (c) **Extra credit** Does the Lagrange problem have a maximum value?
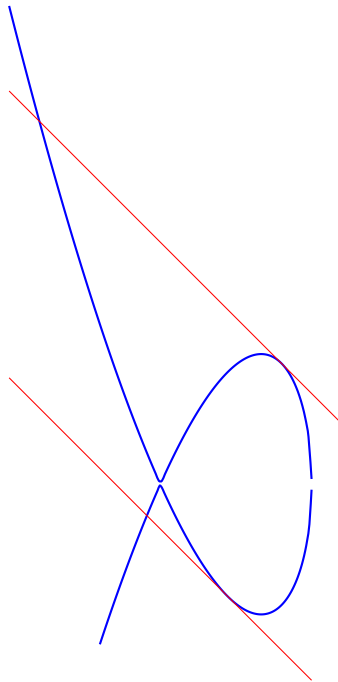
Question 7.

Extra credit

Find a polynomial $p(x)$ with $x = \sqrt[3]{7 + \sqrt{50}} + \sqrt[3]{7 - \sqrt{50}}$ as a zero, and use this to write x as simple as possible.