Exam	EBA 2910 Mathematics for Business Analytics
Date	December 20th 2021 at 0900 - 1400

The exam paper consists of 15 questions, and 1 question for extra credit. All answers must be justified, and the justification should be based on the theory in the course.

- **The answer paper must be handed in as a pdf file. It must be written by hand.**
- The answer paper must be written and prepared individually. Collaboration with others is not permitted and is considered cheating.
- All answer papers are automatically subjected to plagiarism control. Students may also be called in for an oral consultation as additional verification of an answer paper.

Question 1.
We consider the function $f(x) = e^x + 3\,e^{-x}$.

(a) **(6p)** Determine whether f is a convex or concave function.
(b) **(6p)** Compute $f'(x)$ and find maximum and minimum values of f, if they exist.
(c) **(6p)** Sketch the graph of f. Show calculations you think are important to sketch the graph.

We consider the function $f(x;a) = e^x + a\,e^{-x}$ with parameter a.

(d) **(6p)** Determine all values of a such that $f(x;a) = 4$ has at least two solutions.

Question 2.
Compute these indefinite integrals. Write down which integration methods you use.

(a) **(6p)** $\displaystyle\int \frac{2}{\sqrt{x}}\,\mathrm{d}x$
(b) **(6p)** $\displaystyle\int \frac{12}{4-x^2}\,\mathrm{d}x$
(c) **(6p)** $\displaystyle\int 9\sqrt{x}\ln(\sqrt{x})\,\mathrm{d}x$

Question 3.
We consider the linear system $A \cdot \mathbf{x} = \mathbf{b}$ with parameter a, given by

$$A = \begin{pmatrix} 1 & 2 & 1 & 4 \\ 3 & 7 & 2 & a \\ 5 & 12 & 3 & -3 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 0 \\ 9 \\ 16 \end{pmatrix}$$

(a) **(6p)** Use Gaussian elimination to solve the linear system when $a = 0$. Mark the pivot positions.
(b) **(6p)** Determine all values of a such that the lineære systemet is consistent.
(c) **(6p)** Express the vector $\mathbf{w} = (2,1,0)$ as a linear combination of the four column vectors of A for all values of a where this is possible.

Question 4.
We consider the function defined by $f(x,y) = x^2 + y^2 - x^2y^2$.

(a) **(6p)** Find all stationary points of f, and classify them.
(b) **(6p)** Determine the globale maximum and minimum values of f, if they exist.
(c) **(6p)** Solve the optimization problem: $\min f(x,y) = x^2 + y^2 - x^2y^2$ when $xy = 1$.
(d) **(6p)** Estimate the minimum value of $\min f(x,y) = x^2 + y^2 - x^2y^2$ when $xy = a$.

Question 5.
In the figure below, the blue curve is given by the equation $g(x,y) = a$, and the marked region is given by the inequality $g(x,y) \leq a$. We consider the maximum problem

$$\max f(x,y) = x + y \text{ when } g(x,y) \leq a$$

(a) **(6p)** Show that the maximum problem has a solution that lies on the blue curve.
(b) **Extra credit (6p)** Use the figure to estimate the maximum value. Give reasons for your answer.