Exam	MET 11803 Matematikk
Date	June 3rd 2016 at 0900 - 1400

This exam consists of 16+1 problems (one additional problem is for extra credits, and can be skipped). Each problem has a maximal score of 6p, and 96p (16 solved problems) is marked as 100% score.

You must give reasons for your answers. Precision and clarity will be emphasized when evaluating your answers.

Question 1.
We consider the linear system $A\mathbf{x} = \mathbf{b}$ where the matrix A and the vectors $\mathbf{x}$ and $\mathbf{b}$ are given by

$$A = \begin{pmatrix} 2-s & 3 & 3 \\ 3 & 2-s & 3 \\ 3 & 3 & 2-s \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 3 \\ s+4 \\ 1-2s \end{pmatrix}$$

We consider s as a parameter and x,y,z as variables.

(a) **(6p)** Solve the linear system when $s = 8$. How many degrees of freedom are there?
(b) **(6p)** Compute $|A|$ for a general value of s.
(c) **(6p)** Find A^{-1} when $s = 0$, and use A^{-1} to solve the linear system in this case.
(d) **(6p)** Determine the values of s such that the linear system has exactly one solutions, and find x in these cases.

Question 2.
We consider the function $f(x) = 4 - (x^2 + 3)e^x$.

(6p) Compute $f'(x)$ and $f''(x)$. Is f convex? Is f concave?

Question 3.
Compute the indefinite integrals:

(a) **(6p)** $\displaystyle\int \frac{3x-4}{x^2+x}\,\mathrm{d}x$
(b) **(6p)** $\displaystyle\int 18x^2 \ln(x+1)\,\mathrm{d}x$
(c) **(6p)** $\displaystyle\int e^{\sqrt{x}}\,\mathrm{d}x$

Compute the limit. Explain that it can be interpreted as the area of a region R, and show R in a figure.

(d) **(6p)** $\displaystyle\lim_{b\to\infty} \int_1^b \frac{1}{x^2+x}\,\mathrm{d}x$

Question 4.
We consider the function

$$f(x,y) = \sqrt{36 - x^2 - 4y^2}$$

(a) **(6p)** Compute f'_x and f'_y, and find all stationary points of f.
(b) **(6p)** Classify the stationary points as local maxima, local minima or saddle points.
(c) **(6p)** Find the linear approximation of f at the point $(x,y) = (4,2)$.
(d) **(6p)** Find the maximum and minimum values of f, if they exist.

Question 5.
We consider the Lagrange problem:

$$\max / \min f(x,y) = x + 2y - \sqrt{36 - x^2 - 4y^2} \quad \text{when} \quad x^2 + 4y^2 = 36$$

(a) **(6p)** Find the points of the level curve $x^2 + 4y^2 = 36$ where the tangent has slope $y' = 1/2$.
(b) **(6p)** Sketch $D = \{(x,y) : x^2 + 4y^2 = 36\}$. Is D bounded? Describe the curve geometrically.
(c) **(6p)** Solve the Lagrange problem and find the maximum and minimum values, if they exist.

We change the constraint in the Lagrange problem to $x^2 + 4y^2 \leq 36$.

(d) **Extra credit (6p)** Solve the new optimization problem.