Exam	MET 11803 Matematikk for siviløkonomer
Date	May 30th 2017 at 0900 - 1400

This exam consists of 16+1 problems (one additional problem is for extra credits, and can be skipped). Each problem has a maximal score of 6p, and 96p (16 solved problems) is marked as 100% score.

You must give reasons for your answers. Precision and clarity will be emphasized when evaluating your answers.

Question 1.
We consider the linear system $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{pmatrix} 1+a & 2 & 1-a \\ 2 & 1+a & 2 \\ 1-a & 2 & 1+a \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 3+a \\ a^2 \\ 3-a \end{pmatrix}$$

We consider a as a parameter and x,y,z as variables.

(a) **(6p)** Use Gaussian elimination to solve the linear system when $a = 1$.
(b) **(6p)** Find A^{-1} when $a = 1$, and use A^{-1} to solve the linear system in this case.
(c) **(6p)** Determine the values of a such that the linear system has exactly one solution.
(d) **(6p)** Determine the values of a such that the linear system is inconsistent.

Question 2.
Compute the integrals:

$$\text{(a) } \mathbf{(6p)} \int_0^1 \frac{3}{(2-x)^4}\,\mathrm{d}x \qquad \text{(b) } \mathbf{(6p)} \int \frac{2e^x}{e^x + e^{-x}}\,\mathrm{d}x \qquad \text{(c) } \mathbf{(6p)} \int \frac{\ln x}{x^2}\,\mathrm{d}x$$

Question 3.
We consider the function $f(x) = \dfrac{\ln x}{x^2}$ defined for $x > 0$.

(a) **(6p)** Find the maximum and minimum value of f, if they exist.
(b) **(6p)** Make a rough sketch of the area R in the first quadrant bounded by the graph of f, the x-axis, and the line $x = 1$, and compute the area of R.

Question 4.
We consider the function

$$f(x,y) = \frac{2x + 3y - 6}{xy}$$

(a) **(6p)** Compute f'_x og f'_y, and find the stationary points of f.
(b) **(6p)** Classify the stationary points as locale maxima, local minima or saddle points.
(c) **(6p)** Find the maximum and minimum value of f, if they exist.
(d) **(6p)** Show that the level curve $f(x,y) = 5$ intersects the line $y = 1$ in one point, and find the tangent to the level curve at this point.

Question 5.
We consider the Lagrange problem

$$\max / \min f(x,y) = x^4 + 2x^2y^2 - 4y^3 \quad \text{when} \quad x^2 + y^2 = 25$$

(a) **(6p)** Make a sketch of the curve given by $x^2 + y^2 = 25$, and determine if this is a bounded set.
(b) **(6p)** Write down the Lagrange conditions, and find all $(x,y;\lambda)$ that satisfy these conditions.
(c) **(6p)** Solve the Lagrange problem.

We change the constraint in the Lagrange problem to $x^2 + y^2 = 26$.

(d) **Extra credit (6p)** Estimate the maximum and minimum value of the new Lagrange problem.