Eksamen	MET 11803 Matematikk for sivilkonomer
Dato	May 30th 2018 at 0900 - 1400

This exam consists of 16+1 problems (one additional problem is for extra credits, and can be skipped). Each problem has a maximal score of 6p, and 96p (16 solved problems) is marked as 100% score.

You must give reasons for your answers. Precision and clarity will be emphasized when evaluating your answers.

Question 1.
We consider the function given by $f(x) = 0{,}60 \ln(1+x) + 0{,}40 \ln(1-x)$, defined for $0 \leq x < 1$.

(a) **(6p)** Find the maximum point $x = x^*$ and the maximum value $y = f(x^*)$ of f.
(b) **(6p)** Determine whether f is convex or concave.
(c) **(6p)** Show that $f(x) < 0$ when $x > 2x^*$.
(d) **(6p)** Sketch the graph of f.

Question 2.
Compute these integrals:

(a) **(6p)** $\displaystyle\int x\sqrt{x}\, \mathrm{d}x$
(b) **(6p)** $\displaystyle\int \frac{2x-3}{x^2-3x-4}\, \mathrm{d}x$
(c) **(6p)** $\displaystyle\int \ln\sqrt{x}\, \mathrm{d}x$

Question 3.
We consider the linear system $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{pmatrix} 2 & a & 0 \\ a & 1 & a \\ 0 & a & 2 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 3 \\ a \\ -a \end{pmatrix}$$

and a is a parameter.

(a) **(6p)** Solve the linear systemet when $a = 1$.
(b) **(6p)** Find the determinant $\det(A)$, and determine the values of a such that $\det(A) = 0$.
(c) **(6p)** Determine all values of a such that $A \cdot \mathbf{x} = \mathbf{b}$ has infinitely many solutions.
(d) **(6p)** Compute the matrix product $\mathbf{x}^T \cdot A \cdot \mathbf{x}$ when $a = 1$.

Question 4.
We consider the function $f(x,y) = (x-y)\, e^{2xy}$.

(a) **(6p)** Compute the partial derivatives of f, and find all stationary points.
(b) **(6p)** Solve the optimization problem $\max f(x,y)$ when $0 \leq x \leq 1$ and $0 \leq y \leq 1$.

Question 5.
We consider the Lagrange problem

$$\min f(x,y) = x^2 + y^2 - 4y \quad \text{when} \quad 2x + y^2 = -1$$

(a) **(6p)** Sketch the curve $2x + y^2 = -1$, and determine whether this is a bounded set.
(b) **(6p)** Write down the Lagrange conditions, and find all points $(x,y;\lambda)$ that satisfy them.
(c) **(6p)** Solve the Lagrange problem and find the minimum value, if it exists.

Question 6.

Let p be a parameter with $0 < p < 1$, let $q = 1 - p$, and let a,$b > 0$ be positive parameters such that $ap - bq > 0$. We consider the function f given by

$$f(x) = p \ln(1 + ax) + q \ln(1 - bx), \quad 0 \leq x < 1$$

Extra credit (6p) Find the maximum point $x = x^*$ of f, and explain what x^* represents in the optimization problem when the function f and its parameters are interpreted as specified below.

Interpretation of the function f:
We participate in a game where we win a times our bet with probability p, and loose b times our bet with probability q. If we participate in this game n times, with independent outcomes, and each time bet the share x of our capital, then the starting capital X_0 will grow to X_n, which is a stochastic variable with the property that

$$f(x) = E\left[\ln \left(X_n/X_0\right)^{1/n}\right]$$