Exam	MET 11807 Mathematics
Date	May 29th 2019 at 0900 - 1400

This exam consists of 15+1 problems (one additional problem is for extra credits, and can be skipped). Each problem has a maximal score of 6p, and 90p (15 solved problems) is marked as 100% score.

You must give reasons for your answers. Precision and clarity will be emphasized when evaluating your answers.

Question 1.

We consider the linear system $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{pmatrix} 1 & a & 4 \\ 2a & 8 & 12 \\ 5 & 10 & 16 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 11 \\ 40 \\ 51 \end{pmatrix}$$

We consider a as a parameter and x,y,z as variables.

a) **(6p)** Use Gaussian elimination to solve the linear system when $a = 2$. Mark the pivot positions.

b) **(6p)** Compute $\det(A)$, and determine all values of a such that $\det(A) = 0$.

c) **(6p)** Find A^{-1} when $a = 3$.

d) **(6p)** Show that $A^7 \cdot \mathbf{x} = \mathbf{b}$ has exactly one solution for $a = -1$, and express the solution $\mathbf{x}$ in terms of A and $\mathbf{b}$. It is not necessary to compute the solution $\mathbf{x}$ explicitly.

Question 2.

We consider the function defined by $f(x) = \dfrac{x^3}{1-x^2}$.

a) **(6p)** Find the asymptotes of f.

b) **(6p)** Compute $f'(x)$, and determine when f is decreasing.

Question 3.

Compute these integrals:

a) **(6p)** $\displaystyle\int 30(1-x)^5 \, \mathrm{d}x$

b) **(6p)** $\displaystyle\int \frac{12}{4-9x^2} \, \mathrm{d}x$

c) **(6p)** $\displaystyle\int \frac{2e^x}{e^x - e^{-x}} \, \mathrm{d}x$

Question 4.

We buy a building to rent it out. We model the net rental income as a continuous cash flow, such that the net rental income after t years is given by $I(t) = 12\,e^{0.07t}$ (in MNOK per year). We use continuous discounting to compute present values, with discount rate $r = 10\%$.

a) **(6p)** Find the present value of the cash flow we receive by renting out the building for all years to come.

b) **(6p)** We consider selling the property after 7 years if the present value of the selling price is at least as big as the present value of the future cash flow from renting it out. What is the minimal selling price we need to consider selling?

Question 5.
We consider the function defined by $f(x,y) = y^2 - x^3 + 3x$, and call the level curve of f through the point $(x,y) = (-1,2)$ for C.

a) **(6p)** Find all stationary points of f, and classify these points.

b) **(6p)** Find the tangent of C in the point $(x,y) = (-1,2)$. Does the tangent intersect C in any other points?

c) **(6p)** Sketch the curve in the xy-plane given by $4x^2 + y^2 = 4$. What kind of curve is this? Is it bounded?

d) **(6p)** Solve the optimization problem: $\max f(x,y) = y^2 - x^3 + 3x$ when $4x^2 + y^2 = 4$

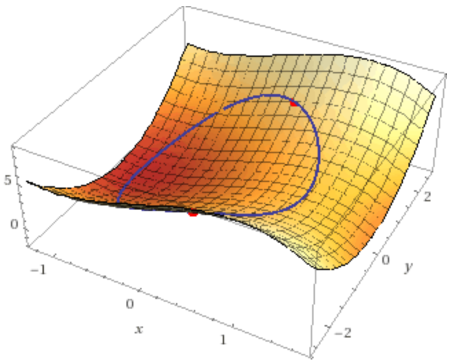

FIGURE 1. Illustration for Question 5

Question 6.
Extra credit (6p) Solve the Lagrange problem: $\min f(x,y) = x$ when $y^2 - x^3 + 3x = 2$