Key Problems

Problem 1.
Compute the indefinite integrals:

a) $\displaystyle\int \ln(x)\ \mathrm{d}x$

b) $\displaystyle\int \ln(x+1)\ \mathrm{d}x$

c) $\displaystyle\int \log_3(x)\ \mathrm{d}x$

Problem 2.
Compute the indefinite integrals:

a) $\displaystyle\int x\ln(x)\ \mathrm{d}x$

b) $\displaystyle\int x^2\ln(x)\ \mathrm{d}x$

c) $\displaystyle\int \sqrt{x}\ln(x)\ \mathrm{d}x$

d) $\displaystyle\int x\sqrt{x}\ln(x)\ \mathrm{d}x$

e) $\displaystyle\int \frac{\ln(x)}{x^2}\ \mathrm{d}x$

f) $\displaystyle\int \frac{\ln(x)}{\sqrt{x}}\ \mathrm{d}x$

g) $\displaystyle\int \frac{\ln(x)}{x\sqrt{x}}\ \mathrm{d}x$

h) $\displaystyle\int \frac{\ln(x)}{x}\ \mathrm{d}x$

Problem 3.
Compute the indefinite integrals:

a) $\displaystyle\int xe^x\ \mathrm{d}x$

b) $\displaystyle\int xe^{-x}\ \mathrm{d}x$

c) $\displaystyle\int (x+1)e^x\ \mathrm{d}x$

d) $\displaystyle\int x^2 e^x\ \mathrm{d}x$

Problem 4.
Compute the indefinite integrals:

a) $\displaystyle\int \frac{4}{4-x}\ \mathrm{d}x$

b) $\displaystyle\int \frac{4}{4-x^2}\ \mathrm{d}x$

c) $\displaystyle\int \frac{4x}{4-x^2}\ \mathrm{d}x$

d) $\displaystyle\int \frac{x^2}{4-x^2}\ \mathrm{d}x$

Problem 5.
Compute the indefinite integrals:

a) $\displaystyle\int \frac{1}{1-x^2}\ \mathrm{d}x$

b) $\displaystyle\int \frac{2x}{1-x^2}\ \mathrm{d}x$

c) $\displaystyle\int \frac{x^2}{1-x^2}\ \mathrm{d}x$

d) $\displaystyle\int \frac{x^2-2x+1}{1-x^2}\ \mathrm{d}x$

e) $\displaystyle\int \frac{1}{(1-x)^2}\ \mathrm{d}x$

f) $\displaystyle\int \frac{2x}{(1-x)^2}\ \mathrm{d}x$

g) $\displaystyle\int \frac{x^2}{(1-x)^2}\ \mathrm{d}x$

h) $\displaystyle\int \frac{x^2-2x+1}{(1-x)^2}\ \mathrm{d}x$

Problem 6.
Compute the indefinite integrals:

a) $\displaystyle\int xe^{1-x^2}\ \mathrm{d}x$

b) $\displaystyle\int x\ln(1-x)\ \mathrm{d}x$

c) $\displaystyle\int \frac{x^3+x^2-2x-6}{x^2-1}\ \mathrm{d}x$

Problem 7.
Compute the indefinite integral:
$$\int 2x^3\,e^{-x^2}\,\mathrm{d}x$$

Problem 8.
Compute the indefinite integral:
$$\int \sqrt{x}\,e^{\sqrt{x}}\,\mathrm{d}x$$

Problem 9.
Compute the indefinite integral:
$$\int \frac{\sqrt{x}+1}{1-\sqrt{x}}\,\mathrm{d}x$$

Problem 10.
We consider the function given by $f(x) = 0{,}60\ln(1+x) + 0{,}40\ln(1-x)$, defined for $0 \le x < 1$.

  a. **(6p)** Find the maximum point $x = x^*$ and the maximal value $y = f(x^*)$ of f.

  b. **(6p)** Determine whether f is convex or concave.

  c. **(6p)** Show that $f(x) < 0$ when $x > 2x^*$.

  d. **(6p)** Sketch the graph of f.

Problem 11.
Problems from the textbook: 9.5.1, 9.6.1 - 9.6.2, 9.6.8 - 9.6.9

Answers to Key Problems

Problem 1.
  a) $x\ln x - x + \mathcal{C}$ b) $(x+1)\ln(x+1) - (x+1) + \mathcal{C}$ c) $(x\ln x - x)/\ln(3) + \mathcal{C}$

Problem 2.
  a) $\frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + \mathcal{C}$ b) $\frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + \mathcal{C}$ c) $\frac{2}{3}x\sqrt{x}\ln x - \frac{4}{9}x\sqrt{x} + \mathcal{C}$

  d) $\frac{2}{5}x^2\sqrt{x}\ln x - \frac{4}{25}x^2\sqrt{x} + \mathcal{C}$ e) $-\frac{1}{x}\ln x - \frac{1}{x} + \mathcal{C}$ f) $2\sqrt{x}\ln x - 4\sqrt{x} + \mathcal{C}$

  g) $-\frac{2}{\sqrt{x}}\ln x - \frac{4}{\sqrt{x}} + \mathcal{C}$ h) $\frac{1}{2}(\ln x)^2 + \mathcal{C}$

Problem 3.
  a) $xe^x - e^x + \mathcal{C}$ b) $-xe^{-x} - e^{-x} + \mathcal{C}$ c) $(x+1)e^x - e^x + \mathcal{C}$ d) $x^2e^x - 2xe^x + 2e^x + \mathcal{C}$

Problem 4.

a) $-4\ln|4-x|+\mathcal{C}$

b) $\ln|2+x|-\ln|2-x|+\mathcal{C} = \ln\left|\dfrac{2+x}{2-x}\right|+\mathcal{C}$

c) $-2\ln|2-x|-2\ln|2+x|+\mathcal{C} = -2\ln|4-x^2|+\mathcal{C}$

d) $-x+\ln|2+x|-\ln|2-x|+\mathcal{C} = -x+\ln\left|\dfrac{2+x}{2-x}\right|+\mathcal{C}$

Problem 5.

a) $\dfrac{1}{2}\ln\left|\dfrac{1+x}{1-x}\right|+\mathcal{C}$

b) $-\ln|1-x^2|+\mathcal{C}$

c) $-x+\dfrac{1}{2}\ln\left|\dfrac{1+x}{1-x}\right|+\mathcal{C}$

d) $-x+\ln\left|\dfrac{1+x}{1-x}\right|+\ln|1-x^2|+\mathcal{C} = -x+2\ln|1+x|+\mathcal{C}$

e) $\dfrac{1}{1-x}+\mathcal{C}$

f) $2\ln|1-x|+\dfrac{2}{1-x}+\mathcal{C}$

g) $x+2\ln|1-x|+\dfrac{1}{1-x}+\mathcal{C}$

h) $x+\mathcal{C}$

Problem 6.

a) $-\frac{1}{2}e^{1-x^2}+\mathcal{C}$

b) $\frac{1}{2}x^2\ln(1-x)-\frac{1}{2}x-\frac{1}{4}x^2-\frac{1}{2}\ln(1-x)+\mathcal{C}$

c) $\dfrac{1}{2}x^2+x-3\ln|x-1|+2\ln|x+1|+\mathcal{C} = \dfrac{1}{2}x^2+x+\ln\left|\dfrac{(x+1)^2}{(x-1)^3}\right|+\mathcal{C}$

Problem 7.

$-x^2e^{-x^2}-e^{-x^2}+\mathcal{C}$

Problem 8.

$2xe^{\sqrt{x}}-4\sqrt{x}e^{\sqrt{x}}+4e^{\sqrt{x}}+\mathcal{C}$

Problem 9.

$5-4\sqrt{x}-x-4\ln|1-\sqrt{x}|+\mathcal{C}$

Problem 10.

a. $x^* = 0{,}20$ and $f(x^*) = 0{,}60\ln(1{,}20)+0{,}40\ln(0{,}80) \approx 0{,}0201$

b. f is concave

c. Since f is decreasing for $x > x^* = 0{,}20$ and $f(2x^*) = 0{,}60\ln(1{,}40)+0{,}40\ln(0{,}60) \approx -0{,}0024 < 0$, it follows that $f(x) < 0$ for $x > 2x^* = 0.4$.

Problem 11.

See answers in the textbook.