Key Problems

Problem 1.
Use elementary row operations to find the inverse of A, if it exists. Check your answer by comparing with the determinant and adjungated matrix of A.

a) $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

b) $A = \begin{pmatrix} 1 & 3 & 0 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{pmatrix}$

c) $A = \begin{pmatrix} 1 & 3 & 0 \\ 2 & 1 & 1 \\ 3 & 4 & 1 \end{pmatrix}$

Problem 2.
Let A be a 2×3-matrix.

a) Is A symmetric?

b) Is $A^T A$ symmetric?

c) Compute $A^T A$ when $A = \begin{pmatrix} 1 & -1 & 3 \\ 3 & 3 & 1 \end{pmatrix}$.

Problem 3.
We consider the linear system $A \cdot \mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} a & 1 & a \\ 1 & 2 & 3 \\ a & 3 & 0 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ -a \\ 3-a \end{pmatrix}$$

and a is a parameter.

a) **(6p)** Solve the linear system when $a = 1$.

b) **(6p)** Find the determinant $\det(A)$, and determine all values of a such that $\det(A) = 0$.

c) **(6p)** Determine all values of a such that $A \cdot \mathbf{x} = \mathbf{b}$ has infinitely many solutions.

d) **(6p)** Compute $A^2 - 3A$ when $a = 1$.

Problem 4.
We consider the linear system $A \cdot \mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} 2-s & 3 & 3 \\ 3 & 2-s & 3 \\ 3 & 3 & 2-s \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \text{og} \quad \mathbf{b} = \begin{pmatrix} 3 \\ s+4 \\ 1-2s \end{pmatrix}$$

We consider s as a parameter and x,y,z as variables.

a) **(6p)** Solve the linear system when $s = 8$. How many degrees of freedom are there?

b) **(6p)** Compute $|A|$ for a general value of s.

c) **(6p)** Find A^{-1} when $s = 0$, and use A^{-1} to solve the linear system in this case.

d) **(6p)** Determine all values of s such that the linear system has exactly one solution, and find x in these cases.

Problem 5.
Compute the partial derivatives f'_x og f'_y:

a) $f(x,y) = 2x + 3y$

b) $f(x,y) = x^2 - y$

c) $f(x,y) = 3x^2 + xy - y^2$

d) $f(x,y) = x^3 + 3xy + 2y^3 - 2x$

e) $f(x,y) = x^2 \ln y$

f) $f(x,y) = e^{xy}$

g) $f(x,y) = xe^y - ye^x$

h) $f(x,y) = \sqrt{x^2 + y^2}$

i) $f(x,y) = \ln(x^2 + xy + y^2)$

Problem 6.
Compute the partial derivatives f'_x og f'_y:

a) $f(x,y) = \dfrac{1}{x+y}$

b) $f(x,y) = \dfrac{2x+3y}{xy}$

c) $f(x,y) = \dfrac{xy}{2x-y}$

d) $f(x,y) = \dfrac{1}{x^2+y^2}$

e) $f(x,y) = \dfrac{1}{x} + \dfrac{1}{y}$

f) $f(x,y) = \dfrac{x}{y} - \dfrac{y}{x}$

Answers to Key Problems

Problem 1.

a) $A^{-1} = \frac{1}{3}\begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$

b) $A^{-1} = \frac{1}{5}\begin{pmatrix} 2 & 6 & -3 \\ 1 & -2 & 1 \\ -5 & -5 & 5 \end{pmatrix}$

c) A is not invertible

Problem 2.

a) No

b) Yes

c) $\begin{pmatrix} 10 & 8 & 6 \\ 8 & 10 & 0 \\ 6 & 0 & 10 \end{pmatrix}$

Problem 3.

a) $(x,y,z) = (2,0,-1)$

b) $|A| = -a(2a+3)$, and $|A| = 0$ for $a = 0$ and $a = -3/2$

c) $a = 0$

d) $\begin{pmatrix} 0 & 3 & 1 \\ 3 & 8 & -2 \\ 1 & -2 & 10 \end{pmatrix}$

Problem 4.

a) There is one degree of freedom for $s = 8$, and the solutions are given by $(x,y,z) = (z-2,z-3,z)$ where z is free.

b) $|A| = -s^3 + 6s^2 + 15s + 8$

c) $A^{-1} = \frac{1}{8}\begin{pmatrix} -5 & 3 & 3 \\ 3 & -5 & 3 \\ 3 & 3 & -5 \end{pmatrix}$ and $(x,y,z) = (0,-1,2)$ for $s = 0$.

d) For $s \neq -1, 8$, the system has exactly one solution with x-coordinate $x = 0$.

Problem 5.

a) $f'_x = 2,\ f_y = 3$

b) $f'_x = 2x,\ f_y = -1$

c) $f'_x = 6x + y,\ f_y = x - 2y$

d) $f'_x = 3x^2 + 3y - 2,\ f_y = 3x + 6y^2$

e) $f'_x = 2x \ln y,\ f_y = x^2/y$

f) $f'_x = ye^{xy},\ f_y = xe^{xy}$

g) $f'_x = e^y - ye^x,\ f_y = xe^y - e^x$

h) $f'_x = \dfrac{x}{\sqrt{x^2+y^2}}, \quad f_y = \dfrac{y}{\sqrt{x^2+y^2}}$

i) $f'_x = \dfrac{2x+y}{x^2+xy+y^2}, \quad f_y = \dfrac{x+2y}{x^2+xy+y^2}$

Problem 6.

a) $f'_x = f'_y = -\dfrac{1}{(x+y)^2}$

b) $f'_x = -\dfrac{2}{y^2}, \quad f'_y = -\dfrac{3}{x^2}$

c) $f'_x = \dfrac{-y^2}{(2x-y)^2}, \quad f'_y = \dfrac{2x^2}{(2x-y)^2}$

d) $f'_x = \dfrac{-2x}{(x^2+y^2)^2},\ f'_y = \dfrac{-2y}{(x^2+y^2)^2}$

e) $f'_x = \dfrac{-1}{x^2}, \quad f'_y = \dfrac{-1}{y^2}$

f) $f'_x = \dfrac{1}{y} + \dfrac{y}{x^2}, \quad f'_y = \dfrac{-x}{y^2} - \dfrac{1}{x}$