Key Problems

Problem 1.
Find the natural domain of definition D_f og range R_f of f:

a) $f(x,y) = 2x + 3y$ b) $f(x,y) = \sqrt{x + 3y}$ c) $f(x,y) = (2x - y)^{-3/2}$ d) $f(x,y) = 17x^{1.2}y^{3.4}$

Problem 2.
Find as many vectors as possible that are normal to the vector $\mathbf{v}$:

a) $\mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ b) $\mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ c) $\mathbf{v} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ d) $\mathbf{v} = \begin{pmatrix} 4 \\ 7 \\ -3 \end{pmatrix}$

Problem 3.
Sketch the level curves $f(x,y) = c$ for the given values of c in the same coordinate system:

a) $f(x,y) = 2x + 3y$ og $c = -2, -1, 0, 1, 2$
b) $f(x,y) = x^2 + y^2$ og $c = -2, -1, 0, 1, 2$
c) $f(x,y) = 4x^2 + 9y^2$ og $c = -2, -1, 0, 1, 2$
d) $f(x,y) = x^2 - 2x + 4y^2$ og $c = -2, -1, 0, 1, 2$

Problem 4.
Describe the graph of $f(x,y) = 3x - 4y + 1$ geometrically.

Problem 5.
Course paper: Remember the deadline on Mon 21/03 at 12.00
Optional: Problems from [Eriksen] (norwegian textbook)
Problem 7.1.1 - 7.1.4, 7.2.1 - 7.2.2, 7.3.1 - 7.3.2 (textbook)

Answers to Key Problems

Problem 1.
a) $D_f = \mathbb{R}^2,\ R_f = \mathbb{R}$
b) $D_f = \{(x,y) \in \mathbb{R}^2 : x + 3y \geq 0\},\ R_f = [0,\infty)$
c) $D_f = \{(x,y) \in \mathbb{R}^2 : 2x - y > 0\},\ R_f = (0,\infty)$
d) $D_f = \{(x,y) \in \mathbb{R}^2 : x,y \geq 0\},\ R_f = [0,\infty)$

Problem 2.
All linear combinations of the vectors:

a) $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ b) $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ c) $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$ d) $\begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}, \begin{pmatrix} -7 \\ 4 \\ 0 \end{pmatrix}$

Problem 3.
a) Straight lines
b) Circles for $c > 0$
c) Ellipses for $c > 0$
d) Ellipses with center (1,0) for $c > -1$

Problem 4.
The graph of f is the plane the intersects the z-axis at $z = 1$ and has normal vector $(3, -4, -1)$.