Key Problems

Problem 1.
Assume that the Lagrange problem max $f(x,y)$ when $g(x,y) = 4$ has maximum value $f(1,3) = 12$ at the ordinary candidate point $(x,y;\lambda) = (1,3;2)$. What is the interpretation of $\lambda = 2$? Use this to estimate the maximum value of the Lagrange problem max $f(x,y)$ when $g(x,y) = 3$.

Problem 2.
We consider the function $f(x,y) = x^3y^2 + x^2 - 2x$.

a) Find all stationary points of f and classify them.

b) Determine whether f has a maximum or minimum.

Problem 3.
We consider the function $f(x,y) = x^3y^2 + x^2y - xy + 1$ with domain of definition $D = \{(x,y) : -1 \le x,y \le 1\}$.

a) Sketch D and describe the boundary points.

b) Find all interior stationary points and classify them.

c) Find $f_{\max}$ and $f_{\min}$ if they exist.

Problem 4.
We consider the Lagrange problem: $\max / \min\ f(x,y) = xy \quad \text{when} \quad x^2 + y^2 = 4$

a) Solve the Lagrange conditions (FOC+C) and find the ordinary candidate points.

b) Are there any admissible points with degenerated constraint?

c) Solve the Lagrange problem.

Problem 5.
We consider the curve C with equation $y(x^2 + y^2) = 2(x^2 - y^2)$.

a) Find all points on C with $y = -1$.

b) Find the tangent of C at each point with $y = -1$.

c) Solve the optimization problem: $\max / \min\ f(x,y) = y \quad \text{when} \quad y(x^2 + y^2) = 2(x^2 - y^2)$

Problem 6.
Solve the optimization problem: $\max / \min\ f(x,y) = x^3 + 3xy + y^3 \quad \text{when} \quad xy = 1$

Problem 7. *Exam MET1180 12/2018*
We consider the function defined by $f(x,y) = 1 + x^2 + y^2 + x^2y^2$.

a) Find all stationary points of f.
b) Compute the Hessian of f, and use it to classify the stationary points.
c) Determine whether f has global maximum or minimum values.
d) Solve the Lagrange problem: $\max f(x,y) = x^2 + y^2 + x^2y^2$ when $x^2 + 2y^2 = 5$

Problem 8. *Exam MET1180 12/2017*
We consider the function $f(x,y) = x^2y^2 + xy + x - y$.

a) Compute the first order partial derivatives and the Hessian of f.
b) Show that the level curve $f(x,y) = 2$ intersects the line $y = x$ in two points (a,a) and (b,b).
c) Find the tangent of the level curve $f(x,y) = 2$ at the points (a,a) and (b,b).
d) Find the stationary points of f, and classify them as local maxima, local minima or saddle points.

Problem 9. *Exam MET1180 12/2017*
We consider the Lagrange problem: $\min f(x,y) = xy \quad \text{when} \quad x^2 + 4y^2 = 4$.

a) Sketch the curve $x^2 + 4y^2 = 4$, and determine if it is bounded.
b) Write down the Lagrange conditions, and find all $(x,y;\lambda)$ that satisfy these conditions.
c) Solve the Lagrange problem.
d) Give an interpretation of the Lagrange multiplicator in a Lagrange problem, and use this interpretation to estimate the minimum value of the new Lagrange problem: $\min f(x,y) = xy \quad \text{when} \quad x^2 + 4y^2 = 5$

Answers to Key Problems

Problem 1.
$f_{\max} \approx 12 + (-1) \cdot 2 = 10$

Problem 2.
a) $(1,0)$ local minimum
b) No maximum or minimum

Problem 3.
a) The boundary points are the sides of the square.
b) $(0,0)$ saddle point
c) $f_{\max} = 2$, $f_{\min} = -2$

Problem 4.
a) $(\pm\sqrt{2}, \pm\sqrt{2}; 1/2)$, $(\pm\sqrt{2}, \mp\sqrt{2}; -1/2)$
b) No
c) $f_{\max} = 2$, $f_{\min} = -2$

Problem 5.
a) $(\pm\sqrt{1/3}, -1)$
b) $y = 2 \mp 3\sqrt{3}x$
c) $f_{\min} = -2$, no maximum value

Problem 6.
No maximum or minimum value.