FORELESNING 1

ELE3717

MATEMATIKK

EIVIND ERIKSEN

08 JAN 2013

Kurset består av tre deler:

a) Sannsynlighetsregning — [R] Ross
b) Matrisemetoder
   - Kvadratiske former
c) Differensiallikninger og Kontrollteori

([FMEA] Sydsæter et al. Further Math...)
Notater på It's Learning.

Eksamen:

5t. Hjelpemidler: Kun kalkulator

Viktig informasjon underveis i kurset finnes i It's Learning:

* Forelesningsplan m/ forelesningsnotater, oppgaver, temaer for hver forelesning
* Litteratur: lærebøker + notater som kan lastes ned

Matematiske modeller:

Matematisk modell

$p = \ldots$

det som vi ønsker å studere

p (pris), ...

predikering

utlede konsekvenser av modellen

Stemmer disse konsekvensene overens med det vi ønsker å studere?

modell

"virkelighet"

① Sannsynlighetsregning

[R] 1.1–1.3

Eks:

Vi kaster en terning. ← Stokastisk forsøk

(stokastisk = tilfeldig)

Mulige resultat: 1, 2, 3, 4, 5, 6 ← Utfallsrom

$S = \{1, 2, 3, 4, 5, 6\}$

Hva er sannsynligheten for å få 1, 2, eller 5? ← Hendelse

$\{1, 2, 5\}$

2? $\{2\}$

Sannsynligheten for å få $\{1,2,5\}$: $p(\{1,2,5\}) = \frac{3}{6} = \underline{\frac{1}{2}}$

$p(2) = \underline{\frac{1}{6}}$

Sannsynlighet p

Defn:

Et stokastisk forsøk er et forsøk der de mulige utfallene er kjente, men faktisk utfall er ukjent.

Utfallsrommet til et stokastisk forsøk er mengden av mulige utfall.

En hendelse er en delmengde av utfallsrommet.

Et sannsynlighetsmål er en funksjon p definert på hendelser med verdier blant tallene i $\mathbb{R}$ (reelle tall);

Dvs: I et stokastisk forsøk med utfallsrom S

For hver hendelse E (så E er en delmengde av S)
så gir sannsynlighetsmålet p en verdi

$$p(E) \in \mathbb{R}$$

Et sannsynlighetsmål skal oppfylle følgende:

i) $0 \le p(E) \le 1$ for alle hendelser E

ii) $p(S) = 1$

iii) $p(E_1 \cup E_2 \cup \dots) = p(E_1) + p(E_2) + \dots$
når $E_1, E_2, \dots$ er parvis disjunkte hendelser

E_1, E_2 er delmengder av S: De er disjunkte
hvis ingen utfall er med i både E_1 og E_2.

$E_1, E_2, E_3, \dots$ er parvis disjunkte hvis to vilkårlige
blant hendelsene er disjunkte, dvs $\left\{ \begin{array}{l} E_1 \text{ og } E_2 \\ E_1 \text{ og } E_3 \\ \text{etc} \end{array} \right.$ er disjunkte.

Union: $E_1 \cup E_2$ = mengden av utfall med i E_1 eller
i E_2 (eller i E_1 og E_2)

Snitt: $E_1 \cap E_2$ = mengden av utfall med i E_1 og
E_2

Eks: $E_1 = \{1, 2\}$ $E_2 = \{3, 4\}$

$E_1 \cup E_2 = \{1, 2, 3, 4\}$

$E_1 \cap E_2 = \emptyset$ (tom mengde)

$\Rightarrow E_1, E_2$ disjunkte

iii)

$p(E_1 \cup E_2) = p(E_1) + p(E_2)$

$= \frac{2}{6} + \frac{2}{6} = \frac{4}{6}$

The probability attributed by an individual to an event is revealed by the conditions under which he would be disposed to bet on that event.

B. de Finetti

Probability does not exist.

B. de Finetti

Personalist views hold that probability measures the confidence that a particular individual has in the truth of a particular proposition.

L. Savage

The probability of an outcome is our estimate for the most likely fraction of a number of repeated observations that will yield that outcome.

R. Feynman

It is likely that the word 'probability' is used by logicians in one sense and by statisticians in another.

F. P. Ramsey

All possible definitions of probability fall short of the actual practice.

W. Feller

1.11 APPENDIX II. REVIEW OF SETS AND FUNCTIONS

It is difficult to make progress in any branch of mathematics without using the ideas and notation of sets and functions. Indeed it would be perverse to try to do so, since these ideas and notation are very helpful in guiding our intuition and solving problems. (Conversely, almost the whole of mathematics can be constructed from these few simple concepts.) We therefore give a brief synopsis of what we need here, for completeness, although it is very likely that the reader will be familiar with all this already.

Sets

A *set* is a collection of things that are called the elements of the set. The elements can be any kind of entity: numbers, people, poems, blueberries, points, lines, and so on, endlessly.

For clarity, upper case letters are always used to denote sets. If the set S includes some element denoted by x, then we say x belongs to S, and write $x \in S$. If x does not belong to S, then we write $x \notin S$.

There are essentially two ways of defining a set, either by a *list* or by a *rule*.

Example 1.11.1. If S is the set of numbers shown by a conventional die, then the *rule* is that S comprises the integers lying between 1 and 6 inclusive. This may be written formally as follows:

$$S = \{x: 1 \leqslant x \leqslant 6 \text{ and } x \text{ is an integer}\}.$$

Alternatively S may be given as a *list*:

$$S = \{1, 2, 3, 4, 5, 6\}. \qquad \bigcirc$$

One important special case arises when the rule is impossible; for example, consider the set of elephants playing football on Mars. This is impossible (there is no pitch on Mars) and the set therefore is empty; we denote the empty set by $\varnothing$. We may write $\varnothing$ as $\{\}$.

If S and T are two sets such that every element of S is also an element of T, then we say that T includes S, and write either $S \subseteq T$ or $S \subset T$. If $S \subset T$ and $T \subset S$ then S and T are said to be equal, and we write $S = T$.

Note that $\varnothing \subset S$ for every S. Note also that some books use the symbol '$\subseteq$' to denote inclusion and reserve '$\subset$' to denote strict inclusion, that is to say, $S \subset T$ if every element of S is in T, and some element of T is not in S. We do not make this distinction.

Combining sets

Given any non-empty set, we can divide it up, and given any two sets, we can join them together. These simple observations are important enough to warrant definitions and notation.

Definition. Let A and B be sets. Their *union*, denoted by $A \cup B$, is the set of elements that are in A or B, or in both. Their *intersection*, denoted by $A \cap B$, is the set of elements in both A and B. △

Note that in other books the union may be referred to as the *join* or *sum*; the intersection may be referred to as the *meet* or *product*. We do not use these terms. Note the following.

Definition. If $A \cap B = \varnothing$, then A and B are said to be *disjoint*. △

We can also remove bits of sets, giving rise to set differences, as follows.

Definition. Let A and B be sets. That part of A which is not also in B is denoted by $A \setminus B$, called the *difference* of A from B. Elements which are in A or B but not both, comprise the *symmetric difference*, denoted by $A \,\Delta\, B$. △

Finally we can combine sets in a more complicated way by taking elements in pairs, one from each set.

Definition. Let A and B be sets, and let

$$C = \{(a, b) \colon a \in A, b \in B\}$$

be the set of ordered pairs of elements from A and B. Then C is called the *product* of A and B and denoted by $A \times B$. △

Example 1.11.2. Let A be the interval $[0, a]$ of the x-axis, and B the interval $[0, b]$ of the y-axis. Then $C = A \times B$ is the rectangle of base a and height b with its lower left vertex at the origin, when $a, b > 0$. ○

Venn diagrams

The above ideas are attractively and simply expressed in terms of *Venn diagrams*. These provide very expressive pictures, which are often so clear that they make algebra redundant. See figure 1.10.

In probability problems, all sets of interest A lie in a universal set Ω, so that $A \subset \Omega$ for all A. That part of Ω which is not in A is called the *complement* of A, denoted by A^c. Formally

$$A^c = \Omega \setminus A = \{x \colon x \in \Omega, x \notin A\}.$$

Obviously, from the diagram or by consideration of the elements

$$A \cup A^c = \Omega, \quad A \cap A^c = \varnothing, \quad (A^c)^c = A.$$

Clearly $A \cap B = B \cap A$ and $A \cup B = B \cup A$, but we must be careful when making more intricate combinations of larger numbers of sets. For example, we cannot write down simply $A \cup B \cap C$; this is not well defined because it is not always true that

$$(A \cup B) \cap C = A \cup (B \cap C).$$

Eks: $S = \{1,2,3,4,5,6\}$

$$\left.\begin{array}{ll} p(1) = \frac{1}{10} & p(4) = \frac{2}{10} \\ p(2) = \frac{2}{10} & p(5) = \frac{2}{10} \\ p(3) = \frac{2}{10} & p(6) = \frac{1}{10} \end{array}\right\} \quad \begin{array}{l} p(\{1,2\}) = \frac{3}{10} \\ p(\{1,2,5\}) = \frac{5}{10} \\ \vdots \end{array}$$

Dette er et sannsynlighetsmål: Egenskap i) - iii) er oppfylt.

Krav:

i) $0 \leq p(E) \leq 1$ for alle hendelser E

ii) $p(S) = 1$

iii) $p(E_1 \cup E_2 \cup \ldots) = p(E_1) + p(E_2) + \ldots$ når $E_1, E_2, \ldots$ er parvis disjunkte hendelser

Konsekvenser:

E^c = alle utfallene som ikke er i E (Komplement)

a) $p(E^c) = 1 - p(E)$

b) $p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$

c) Hvis $E = \{s_1, s_2, \ldots, s_r\}$, så er $p(E) = p(s_1) + p(s_2) + \ldots + p(s_r)$.

Bevis:

a) $E \cup E^c = S$ $\Rightarrow$ $p(E) + p(E^c) = p(S) = 1$

$E \cap E^c = \emptyset$

$$\Updownarrow$$

$$\underline{\underline{p(E^c) = 1 - p(E)}}$$

Oppgave: Bevis b) - c)!

Eks: Vi kaster to terninger

E = summen av terningene er seks

$p(E) = ?$

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), \ldots, \ldots, (6,4), (6,5), (6,6)\}$$

$|S| = 36$ (antall utfall)

$E = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$ $\quad |E| = 5$

$$p(E) = \underline{\underline{\frac{5}{36}}}$$

Generelt:

Når alle utfall i S er like sannsynlige, så er

$$p(E) = \frac{|E|}{|S|} = \frac{\text{antall gunstige utfall}}{\text{antall mulige utfall}}$$

Eks: Vi kaster en mynt helt vi får kron.
Hva er sannsynlighet for at vi må kaste 3 ganger?

$$S = \{K, MK, MMK, MMMK, \ldots\}$$

$$E = \{MMK\}$$

$$p(E) = \cancel{8}\ \underline{\underline{1/8}} \quad \text{siden} \quad \begin{cases} p(K) = 1/2 \\ p(MK) = 1/4 \\ p(MMK) = 1/8 \\ \vdots \\ p(\underbrace{MM\cdots M}_{n-1}K) = \frac{1}{2^n} \end{cases}$$

$$\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n} + \cdots = \frac{1/2}{1 - 1/2} = \frac{1/2}{1/2} = \underline{1} \quad \text{ok} \right)$$

Ordliste:

sample space = utfallsrom
event = hendelse
probability = sannsynlighet
set = mengde
subset = delmengde
intersection = snitt