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① Repetesjon: Kont. stokastisk variabel X

- Sannsynlighetstetthet: $f(x) = f_X(x)$

$$P(a \le X \le b) = \int_a^b f(x)\,dx$$

Krav: i) $f(x) \ge 0$

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

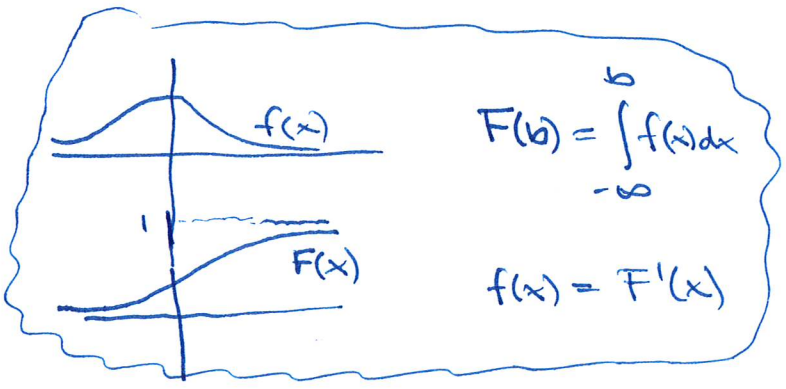
- Kumulativ fordelingsfunksjon: $F(x) = F_X(x)$

$$P(X \le b) = F(b)$$

Krav: i) $F(x)$ voksende funksjon

ii) $\lim_{x \to \infty} F(x) = 1$

$\lim_{x \to -\infty} F(x) = 0$



$$F(b) = \int_{-\infty}^{b} f(x)\,dx$$

$$f(x) = F'(x)$$

Forventning og varians:

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx \qquad E[h(X)] = \int_{-\infty}^{\infty} h(x) \cdot f(x)\,dx$$

$$Var(X) = E[(X-\mu)^2] = E(X^2) - E(X)^2 = E(X^2) - \mu^2 \geq 0$$

$\mu = E(X)$

(2) Dobbeltintegral:

Eks:
$$\int_1^2 \int_2^3 xy^2\,dy\,dx = \int_{x=1}^{x=2} \int_{y=2}^{y=3} xy^2\,dy\,dx$$

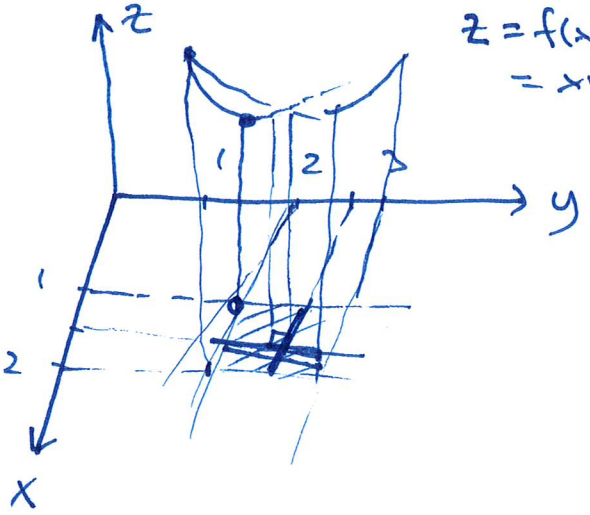
$$\int_2^3 xy^2\,dy = x\int_2^3 y^2\,dy = x\cdot\left[\tfrac{1}{3}y^3\right]_2^3$$

$$= x\left(\tfrac{1}{3}\cdot 3^3\right) - x\left(\tfrac{1}{3}\cdot 2^3\right) = 9x - \tfrac{8}{3}x = \tfrac{27}{3}x - \tfrac{8}{3}x$$

$$= \tfrac{19}{3}x$$

$$= \int_1^2 \tfrac{19}{3}x\,dx = \left[\tfrac{19}{3}\cdot\tfrac{1}{2}x^2\right]_1^2$$

$$= \tfrac{19}{6}\cdot 2^2 - \tfrac{19}{6}\cdot 1^2 = \tfrac{4\cdot 19}{6} - \tfrac{19}{6} = \tfrac{3\cdot 19}{6_2} = \tfrac{19}{2}$$



$z = f(x,y) = xy^2$

$\int_1^2 \int_2^3 xy^2 \, dy dx$ = volum under grafen til f på området

$\{1 \le x \le 2, \ 2 \le y \le 3\}$

Når $f(x,y) \ge 0$ for $a \le x \le b$, $c \le y \le d$, så er

$$\int_a^b \int_c^d f(x,y) \, dy \, dx = \text{volumet under grafen } z = f(x,y) \text{ på rektangelet } a \le x \le b, \ c \le y \le d.$$

Eks:

$$\int_1^2 \int_2^3 xy^2 \, dy dx = \int_2^3 \int_1^2 xy^2 \, dx \, dy = \int_2^3 \left[\tfrac{1}{2} x^2 y^2 \right]_{x=1}^{x=2} dy$$

$$= \int_2^3 \tfrac{1}{2} \cdot 2^2 y^2 - \tfrac{1}{2} \cdot 1^2 y^2 \, dy = \int_2^3 2y^2 - \tfrac{1}{2} y^2 \, dy = \int_2^3 \tfrac{3}{2} y^2 \, dy$$

$$= \left[\tfrac{3}{2} \cdot \tfrac{1}{3} y^3 \right]_2^3 = \tfrac{1}{2} \cdot 3^3 - \tfrac{1}{2} \cdot 2^3 = \tfrac{27}{2} - \tfrac{8}{2} = \tfrac{19}{2}$$

③ Simultane fordelinger

X, Y kont. stokastiske variabler simultant fordelte

dvs X, Y har verdier som avhenger av utfall av det samme stokastiske forsøket

Simultan tetthetsfunksjon: $f(x,y)$

slik at $p(a \le X \le b, c \le Y \le d) = \int_a^b \int_c^d f(x,y)\, dy\, dx$

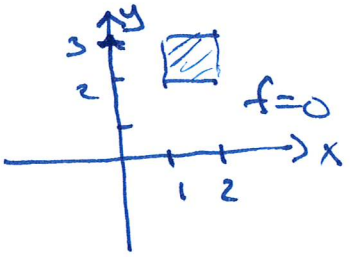
Krav:

i) $f(x,y) \ge 0$ for alle x, y

ii) $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)\, dy\, dx = 1$

Eks:

$$f(x,y) = \begin{cases} xy^2, & \text{for } 1 \le x \le 2,\ 2 \le y \le 3 \\ 0, & \text{ellers} \end{cases}$$



$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)\, dy\, dx = \int_1^2 \int_2^3 xy^2\, dy\, dx = \frac{19}{2}$$

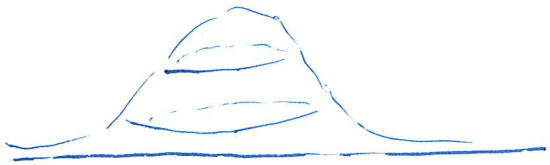
← regnet ut dette dobbeltint. tidligere

nei! ikke en simultan tetthetsfn.

Men:

$$f(x,y) = \begin{cases} \frac{2}{19} xy^2, & 1 \le x \le 2,\ 2 \le y \le 3 \\ 0, & \text{ellers} \end{cases}$$

er en simultan tetthetsfn.

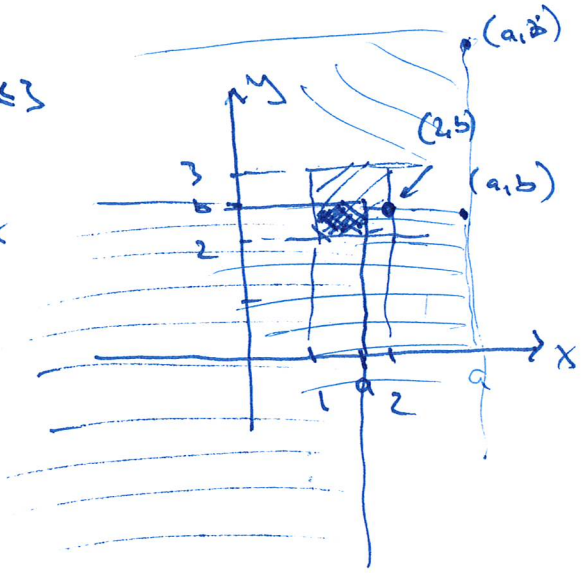


← bivariat normalfordeling

Kumulativ sannsynlighetsfordeling:

$$F_{X,Y}(a,b) = P(X \le a, Y \le b) = \int_{-\infty}^{a}\int_{-\infty}^{b} f(x,y)\,dy\,dx$$

Eks: $f(x,y) = \frac{2}{19}xy^2$, $1 \le x \le 2, 2 \le y \le 3$



i) $1 \le a \le 2$, $2 \le b \le 3$ $\quad F(a,b) = \int_{-\infty}^{a}\int_{-\infty}^{b} f(x,y)\,dy\,dx$

$$= \int_{1}^{a}\int_{2}^{b} \frac{2}{19}xy^2\,dy\,dx$$

$$= \int_{1}^{a} \frac{2}{19}x\left[\frac{1}{3}y^3\right]_2^b dx$$

$$= \int_{1}^{a} \frac{2}{19}x\left(\frac{1}{3}b^3 - \frac{8}{3}\right)dx = \frac{2}{19}\cdot\frac{1}{3}\left[\frac{1}{2}x^2(b^3-8)\right]_1^a$$

$$= \frac{2}{19\cdot 3}(b^3-8)\cdot\frac{1}{2}\left(a^2-1\right) = \frac{(a^2-1)(b^3-8)}{57}$$

ii) $a > 2$, $2 \le b \le 3$: $\quad F(a,b) = F(2,b) = \frac{(2^2-1)(b^3-8)}{57} = \frac{1}{19}(b^3-8)$

iii) $a > 2, b > 3$: $\quad F(a,b) = 1$

Faktor: $F''_{xy} = f(x,y)$

Eks: $F(a,b) = p(X \le a, Y \le b) = \frac{(a^2-1)(b^3-8)}{57}$ når $1 \le a \le 2$, $2 \le b \le 3$

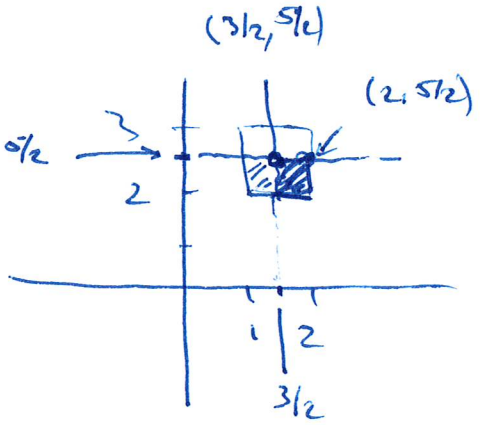
$F(x,y) = \frac{(x^2-1)(y^3-8)}{57}$ når $1 \le x \le 2$, $2 \le y \le 3$

$F'_x = \frac{y^3-8}{57} \cdot 2x$

$F''_{xy} = \left(\frac{y^3-8}{57} \cdot 2x\right)'_y = \frac{2x}{57} \cdot 3y^2 = \frac{2 \cdot 3}{19 \cdot 57} xy^2 = \frac{2}{19} xy^2$

Eks: $p(X \ge 3/2, Y \le 5/2)$

$= \int_{3/2}^{2} \int_{2}^{5/2} \frac{2}{19} xy^2 \, dy \, dx$



$F(2, 5/2) - F(3/2, 5/2)$

$= \frac{(2^2-1)((5/2)^3-8)}{57} - \frac{((3/2)^2-1)\cdot((5/2)^3-8)}{57} = \frac{\frac{125}{8}-8}{57}\left(3 - \frac{5}{4}\right)$

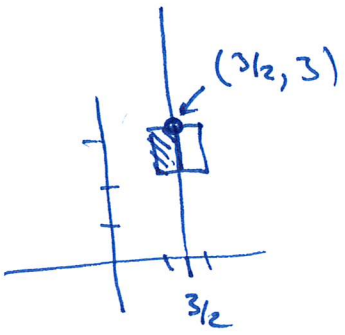
$= \frac{125-64}{8 \cdot 57} \cdot \frac{7}{4} = \frac{61 \cdot 7}{32 \cdot 57} = 0.2341 \simeq 0.23$

(4) Marginaler:

Anta at X og Y er simultant fordelte kont. stokastiske variabler med tetthetsfu. $f(x,y)$

$$P(X \le a) = P(X \le a, Y \le \infty) = \int_{-\infty}^{a}\int_{-\infty}^{\infty} f(x,y)\,dy\,dx$$

Eks: $f(x,y) = \frac{2}{19}xy^2 \quad 1 \le x \le 2,\ 2 \le y \le 3$



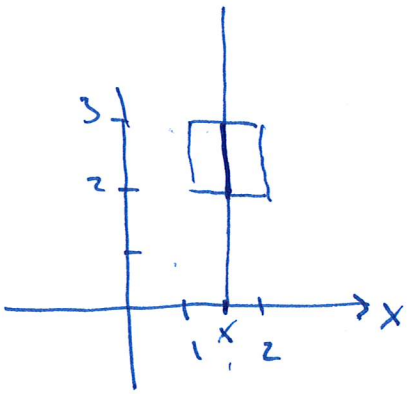
$$P(X \le 3/2) = \int_{1}^{3/2}\int_{2}^{3} \frac{2}{19}xy^2\,dy\,dx = F(3/2, 3)$$

$f_X(x)$

$$= \frac{5/4 \cdot 19}{57} = \frac{5}{12}$$

Marginal tetthetsfunksjon for X:

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy \leftarrow \text{tetthetsfu. for } X$$



Eks: $f(x,y) = \frac{2}{19}xy^2 \quad 1 \le x \le 2,\ 2 \le y \le 3$

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_{2}^{3} \frac{2}{19}xy^2\,dy = \frac{2}{19}x\left[\frac{1}{3}y^3\right]_2^3$$

$$= \frac{2}{19 \cdot 3}x(27-8) = \frac{2 \cdot 19}{19 \cdot 3}x = \frac{2}{3}x \quad \text{for } 1 \le x \le 2$$

$$P(X \le 3/2) = \int_{+1}^{3/2} \frac{2}{3}x\,dx = \left[\frac{2}{3}\cdot\frac{1}{2}x^2\right]_1^{3/2} = \frac{1}{3}\left(\frac{9}{4} - 1\right) = \frac{1}{3}\cdot\frac{5}{4} = \frac{5}{12}$$

$f_X(x) = \frac{2}{3}x \quad , \quad 1 \le x \le 2$

$$E(X) = \int_{-\infty}^{\infty} x \cdot f_X(x)\,dx = \int_1^2 x \cdot \frac{2}{3}x\,dx = \left[\frac{2}{3}\cdot\frac{1}{3}x^3\right]_1^2$$

$$= \frac{2}{9}\left(2^3 - 1^3\right) = \frac{2\cdot 7}{9} = \underline{\underline{\frac{14}{9}}}$$

$$E(X^2) = \int_1^2 x^2 \cdot \frac{2}{3}x\,dx = \left[\frac{2}{3}\cdot\frac{1}{4}x^4\right]_1^2 = \frac{1}{6}\left(2^4 - 1^4\right) = \underline{\frac{15}{6}}$$

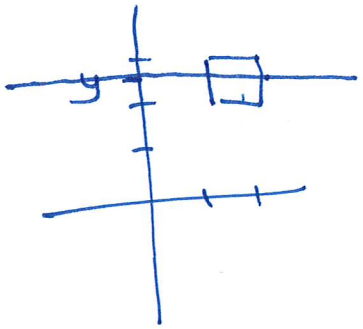
$$Var(X) = E(X^2) - E(X)^2 = \frac{15}{6} - \left(\frac{14}{9}\right)^2 = \frac{15\cdot 9\cdot 3}{6\cdot 9\cdot 3} - \frac{14^2\cdot 2}{9\cdot 9\cdot 2}$$

$$= \frac{405 - 196\cdot 2}{162} = \underline{\underline{\frac{13}{162}}}$$

$14\cdot 14 =$
140
$+\ 56$
196

Marginal tetthetsfunksjon for Y

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx \quad \leftarrow \text{tetthetsfu. for } Y$$



Eks: $f_Y(y) = \int_1^2 \frac{2}{19}xy^2\,dx$

$$= \frac{2}{19}y^2\left[\frac{1}{2}x^2\right]_1^2$$

$$= \frac{2}{19}y^2\left(2 - \tfrac{1}{2}\right) = \frac{\cancel{2}}{19}\cdot\frac{3}{\cancel{2}}y^2 = \underline{\frac{3}{19}y^2}, \quad 2 \le y \le 3$$

$$E(Y) = \int_2^3 y \cdot \underbrace{\frac{3}{19}y^2}_{f_Y(y)}\,dy = \left[\frac{3}{19}\cdot\frac{1}{4}y^4\right]_2^3 = \frac{3}{19\cdot 4}\left(81 - 16\right)$$

$$= \frac{65\cdot 3}{19\cdot 4} = \frac{195}{76} \simeq \underline{2.57}$$

X, Y simultant fordelte kont. stok. variabler med simultan tetthetsfu.

$$f(x,y) = \frac{2}{19}xy^2, \quad 1 \le x \le 2, \quad 2 \le y \le 3$$

mer generelt, mer informasjon

$$E(XY) = ?$$

X kont. stok. var med tetthetsfu.

$$f_X(x) = \frac{2}{3}x, \quad 1 \le x \le 2$$

og

Y kont. stok. var. med tetthetsfu.

$$f_Y(y) = \frac{3}{19}y^2, \quad 2 \le y \le 3$$