Key Problems

Problem 1.
Compute the determinant of these matrices:

a) $A = \begin{pmatrix} 1 & 2 & 5 \\ 3 & 1 & 2 \\ 1 & 2 & 4 \end{pmatrix}$ b) $A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 4 & 2 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 0 & 0 & 1 \end{pmatrix}$ c) $A = \begin{pmatrix} 1 & a & b \\ a & 1 & c \\ b & c & 1 \end{pmatrix}$

Problem 2.
Use minors to determine the rank of these matrices. Can you find a base of the column space $\mathrm{Col}(A)$ based on the minors?

a) $A = \begin{pmatrix} 4 & 1 & 1 & 3 & 7 \\ 2 & 1 & 0 & 1 & 0 \\ 1 & 0 & 3 & 1 & 0 \end{pmatrix}$ b) $A = \begin{pmatrix} 1 & 3 & 2 & 4 \\ 2 & -1 & 7 & 3 \\ 4 & 5 & 11 & 10 \end{pmatrix}$ c) $A = \begin{pmatrix} 3 & 0 & 0 & 7 \\ 0 & 5 & 5 & 0 \\ 0 & 5 & 5 & 0 \end{pmatrix}$

Problem 3.
Use minors to find the rank of these matrices:

a) $A = \begin{pmatrix} 1 & 3 & t \\ 2 & 5 & 7 \end{pmatrix}$ b) $A = \begin{pmatrix} a & 7 & -3 & 5 & 10 \\ 2 & -3 & 1 & 4 & 18 \\ 1 & 24 & -10 & 11 & 12 \end{pmatrix}$ c) $A = \begin{pmatrix} 1 & a & b \\ a & b & 1 \end{pmatrix}$

Problem 4.
Use minors to determine the number of solutions of these linear systems. What are the possible choices of free variables, if any?

a)
$$\begin{array}{rcrcrcr} x & + & y & + & z & = & 6 \\ x & + & 2y & + & tz & = & 13 \\ x & + & 3y & + & 9z & = & 24 \end{array}$$

b)
$$\begin{array}{rcrcrcrcr} x & + & 4y & + & 5z & - & 3w & = & 6 \\ 2x & + & 7y & + & z & & & = & 4 \\ x & + & 5y & + & 4z & - & 8w & = & 1 \end{array}$$

Problems from the Workbook

Exercise problems:	Eriksen [E] 3.1 - 3.15 (see It's Learning)
Optional problems:	Workbook [W] 2.1 - 2.25

Answers to Key Problems

Problem 1.

a) $|A| = 5$ b) $|A| = -96$ c) $|A| = 1 - a^2 - b^2 - c^2 + 2abc$

Problem 2.

The pivot positions are marked in blue, and the corresponding vectors form a base of $\mathrm{Col}(A)$.

a) $\mathrm{rk}\begin{pmatrix} 4 & 1 & 1 & 3 & 7 \\ 2 & 1 & 0 & 1 & 0 \\ 1 & 0 & 3 & 1 & 0 \end{pmatrix} = 3$ b) $\mathrm{rk}\begin{pmatrix} 1 & 3 & 2 & 4 \\ 2 & -1 & 7 & 3 \\ 4 & 5 & 11 & 10 \end{pmatrix} = 3$ c) $\mathrm{rk}\begin{pmatrix} 3 & 0 & 0 & 7 \\ 0 & 5 & 5 & 0 \\ 0 & 5 & 5 & 0 \end{pmatrix} = 2$

Problem 3.

a) $\mathrm{rk}\, A = 2$ for all t b) $\mathrm{rk}\, A = \begin{cases} 2, & a = 1 \\ 3, & a \neq 1 \end{cases}$ c) $\mathrm{rk}\, A = \begin{cases} 1, & (a,b) = (1,1) \\ 2, & (a,b) \neq (1,1) \end{cases}$

Problem 4.

a) One solution if $t \neq 5$, and no solutions if $t = 5$

b) Infinitely many solutions (one degree of freedom). The possible choices for a free variable are x, y, z or w.