Key Problems

Problem 1.
We consider the optimal portfolio problem for the data given as Example 1 of *Notes on optimal portfolio theory* (see page 2 of these notes). Use Python to:

a) Find the minimum variance portfolio $\omega = (\omega_1, \omega_2, \omega_3)$, its expected return μ_M and standard deviation σ_M.

b) For each given $\mu_0 > \mu_M$, write a function that returns the standard deviation of a minimum variance portfolio with $\mu = \mu_0$.

c) Draw the curve consisting of the points (σ_0,μ_0) where $\mu_0 \geq \mu_M$ and σ_0 is the standard deviation of the minimal variance portfolio with $\mu = \mu_0$.

Problem 2.
Solve the optimal control problem

$$\max / \min \int_0^3 \ln(y' + y + e^{-t})\, \mathrm{d}t \text{ when } \begin{cases} y(0) &= 2 \\ y(3) &= 5e^{-3} \end{cases}$$

Is the solution a maximum or a minimum? What is the maximum or minimum value?

Answers to Key Problems

Problem 1.
a) We find $\omega = (0.4411, 0.3656, 0.1933)$, $\sigma_M = 0.07268$ and $\mu_M = 0.02489$

Problem 2.
The function $y^* = (2 + t)e^{-t}$ gives a maximum, with maximum value $3\ln(2) - 9/2 \approx -2.42$.