EXAMINATION QUESTION PAPER - Take-home examination

# ELE 07811
# Mathematics - Elective

Department of Economics

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| **Start date:** | 18.01.2021 | Time 09.00 |
| **Finish date:** | 18.01.2021 | Time 12.00 |
| Weight: | 100% of ELE 0781 | |
| Total no. of pages: | 2 incl. front page | |
| No. of attachments files to question paper: | 0 | |
| To be answered: | Individually | |
| Answer paper size: | No limit. excl. attachments | |
| Max no. of answer paper attachment files: | 0 | |
| Allowed answer paper file types: | pdf | |

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| Exam | Final exam in ELE 3781 Mathematics elective |
| Date | January 18th, 2021 at 0900 - 1200 |

This exam consists of 11 problems. **You must give reasons for all your answers**. To get full score, your answers should be **short, clear, and precise**.

- **You must hand in your exam papers as a single PDF file. It must be handwritten.**
- The answer paper must be written and prepared individually. Collaboration with others is not permitted and is considered cheating.
- All answer papers are automatically subjected to plagiarism control. Students may also be called in for an oral consultation as additional verification of an answer paper.

**Question 1.**
We consider the matrix given by

$$A = \begin{pmatrix} 1 & 2 & 0 & 2 \\ 2 & 4 & 2 & 0 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 6 & 10 \end{pmatrix}$$

(a) **(6p)** Find a base of Null($A$), and determine the eigenvalues of $A$.
(b) **(6p)** Find a base of Col($A$). If possible, find a vector in $\mathbb{R}^4$ not in Col($A$).
(c) **(6p)** Determine the definiteness of the quadratic form $f(\mathbf{x}) = \mathbf{x}^T A\,\mathbf{x}$.
(d) **(6p)** Explain how to use eigenvalues and eigenvectors to find the maximal and minimal value of $f(\mathbf{x}) = \mathbf{x}^T A\,\mathbf{x}$ on $D = \{\mathbf{x} : \mathbf{x}^T\mathbf{x} = 1\}$. You should lay out a procedure for how to do this, and express the answers in terms of eigenvalues and eigenvectors (numerical values are not necessary).

**Question 2.**

(a) **(6p)** Solve $\max f(x, y, z) = \ln(5 - x^2 + xy - y^2 + yz - z^2 - xz)$.
(b) **(6p)** Determine whether $D = \{(x, y, z, w) : xw + yz \leq -2\} \subseteq \mathbb{R}^4$ is a compact set.
(c) **(6p)** Find all points satisfying the Kuhn-Tucker conditions, and the value of the objective function at each of these points:

$$\min f(x, y, z, w) = x^2 + 4y^2 + 9z^2 + w^2 \text{ subject to } xw + yz \leq -2$$

**Question 3.**

(a) **(6p)** Solve the difference equation $y_{t+2} - y_{t+1} - 2y_t = 4t$, and find $y_{17}$ when $y_0 = y_1 = 1$.
(b) **(6p)** Determine whether $t^2y' + 2ty = 1$ is (i) separable, (ii) linear, (iii) exact. Use this to solve the differential equation in at least two different ways.
(c) **(6p)** Find a linear second order differential equation with $y = 3e^{-2t} - 5e^t + 12e^{-3t}$ as solution.
(d) **(6p)** Find a $3 \times 3$ matrix $A$ such that $\mathbf{y}' = A\mathbf{y}$ has a solution $\mathbf{y} = (y, y', y'')$ with $y$ as in (c).