

Non-negative matrices

October 7, 2020

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[1]: import numpy as np

[2]: # Some examples
A = np.array([[0,0,1/3,1/2],[1/2,0,1/3,1/2],[1/2,1,0,0],[0,0,1/3,0]])
B = np.array([[0,3,0,0,1,2],[3,0,2,2,2,1],[6,4,0,2,1,1],[3,1,1,0,2,2],
              [2,1,2,4,0,2],[1,2,2,4,4,0]])

[3]: # Find eigenvalues/eigenvectors of A
eval, evec = np.linalg.eig(A)

[4]: # print eigenvalues of A
eval

[4]: array([ 1.          +0.j          , -0.36062333+0.41097555j,
           -0.36062333-0.41097555j, -0.27875333+0.j          ])

[5]: # dominant eigenvalue of A (has rounding error, should be 1)
dominant = max(abs(eval)).astype(np.complex)
dominant

[5]: (0.9999999999999991+0j)

[6]: # print eigenvectors of A (columns)
evec

[6]: array([[ 0.36050506+0.j          , -0.3583904 +0.07139457j,
           -0.3583904 -0.07139457j, -0.48070923+0.j          ],
          [ 0.54075759+0.j          , -0.09315321+0.2746779j ,
           -0.09315321-0.2746779j ,  0.38153917+0.j          ],
          [ 0.72101012+0.j          ,  0.75521571+0.j          ,
           0.75521571-0.j          , -0.50648562+0.j          ],
          [ 0.24033671+0.j          , -0.3036721 -0.34607247j,
           -0.3036721 +0.34607247j,  0.60565568+0.j          ]])

[7]: # positive eigenvector of A = Frobenius vector
frob = evec[:,eval == dominant].real
frob/frob.sum()
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[7]: array([[0.19354839],
           [0.29032258],
           [0.38709677],
           [0.12903226]])
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[8]: # Test irreducibility of A: If  $A^m > 0$  for some  $m$ , then  $A$  is
# irreducible (with  $k = 1$ ) -> Yes with  $m = 4$ 
np.linalg.matrix_power(A,4)
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[8]: array([[0.16666667, 0.16666667, 0.22222222, 0.20833333],
           [0.29166667, 0.25          , 0.30555556, 0.33333333],
           [0.45833333, 0.41666667, 0.33333333, 0.375        ],
           [0.08333333, 0.16666667, 0.13888889, 0.08333333]])
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[9]: # Find eigenvalues/eigenvectors of B
eval, evec = np.linalg.eig(B)
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[10]: # print eigenvalues of B
eval
```

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[10]: array([ 9.97595547+0.j          , 0.2618527 +0.j          ,
           -2.33052264+2.4535172j, -2.33052264-2.4535172j,
           -2.92699089+0.j          , -2.64977201+0.j          ])
```

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[11]: # dominant eigenvalue of B
dominant = max(abs(eval)).astype(np.complex)
dominant
```

```
[11]: (9.975955472709517+0j)
```

```
[12]: # print eigenvectors of B (columns)
evec
```

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[12]: array([[ -0.25915147+0.j          , -0.02242215+0.j          ,
           -0.28373063+0.39762651j, -0.28373063-0.39762651j,
           -0.19966792+0.j          ,  0.00385305+0.j          ],
           [-0.37981725+0.j          , -0.33041644+0.j          ,
           0.12394064-0.21697628j,  0.12394064+0.21697628j,
           0.43403498+0.j          , -0.31675929+0.j          ],
           [-0.47245877+0.j          , -0.76893151+0.j          ,
           0.55028767+0.j          ,  0.55028767-0.j          ,
           -0.38906304+0.j          ,  0.66324781+0.j          ],
           [-0.3511624 +0.j          ,  0.31514212+0.j          ,
           0.11613241+0.21296665j,  0.11613241-0.21296665j,
           0.60323001+0.j          , -0.49549077+0.j          ],
           [-0.42761827+0.j          ,  0.26375719+0.j          ,
           0.06996176-0.21540723j,  0.06996176+0.21540723j,
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-0.49393815+0.j      ,  0.01482173+0.j      ],
[-0.50910674+0.j      ,  0.36081041+0.j      ,
-0.37806325-0.37823975j, -0.37806325+0.37823975j,
-0.11187031+0.j      ,  0.46262323+0.j      ]])

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[13]: # positive eigenvector of B = Frobenius vector
frob = evec[:,eval == dominant].real
frob/frob.sum()

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[13]: array([[0.10801061],
            [0.15830238],
            [0.19691403],
            [0.14635945],
            [0.17822515],
            [0.21218838]])

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[14]: # Test irreducibility: If  $B^m > 0$  for some  $m$ , then  $B$  is
# irreducible (with  $k = 1$ ) -> Yes for  $m = 2$ 
np.linalg.matrix_power(B,2)

```

```

[14]: array([[13,  5, 12, 18, 14,  5],
            [23, 23,  8, 16, 13, 16],
            [21, 23, 14, 16, 22, 22],
            [15, 19, 10, 20, 14, 12],
            [29, 22, 10, 14, 22, 15],
            [38, 19, 16, 24, 15, 22]])

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