FORK1003 - Final test in linear algebra

4th August 2010

Name and student number:

Problem 1. Compute $-2A + 5B$ when

$$A = \begin{pmatrix} 1 & 3 \\ -2 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & 3 \\ 2 & 1 \end{pmatrix}.$$

Problem 2. Compute AB and BA, if possible, for the following:

(1) $A = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$ and $B = \begin{pmatrix} -3 & 1 & 1 \end{pmatrix}$

(2) $A = \begin{pmatrix} 5 & -3 \\ 10 & 11 \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

(3) $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 2 \\ 3 & 2 \\ -1 & 2 \end{pmatrix}$

Problem 3. Compute the determinants

(a) $\begin{vmatrix} 1 & 2 \\ 4 & 8 \end{vmatrix}$ (b) $\begin{vmatrix} 2 & -13 \\ -2 & 12 \end{vmatrix}$

Problem 4. Write

$$5x_1 - 7x_2 = -2$$
$$7x_1 - 10x_2 = 1$$

as $A\mathbf{x} = \mathbf{b}$. Find A^{-1} and use this to solve the system of equations.

Problem 5. Compute the determinant of A by cofactor expansion along a suitable row and determine if the matrix is invertible.

(a) $A = \begin{pmatrix} 2 & -1 & 2 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$ (b) $A = \begin{pmatrix} 1 & -1 & 0 & 1 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & 3 & -2 \\ -1 & 0 & 0 & 1 \end{pmatrix}$

Problem 6. Use Gauss elimination to solve the following linear system when $r = 1$:

$$x_1 + x_2 + x_3 = 6$$
$$x_1 - 2x_2 + 4x_3 = 3$$
$$x_1 - x_2 + rx_3 = 4$$

Are there any values of r such that the system is inconsistent? Are there any values of r such that the system has infinitely many solutions?

Problem 7. Write the following system of linear equations as $A\mathbf{x} = \mathbf{b}$ and use Cramers rule to find x_2:

$$\begin{aligned} 2x_1 - x_2 + 2x_3 &= 0 \\ x_1 - 2x_2 - x_3 &= 3 \\ x_1 + x_2 - x_3 &= 0 \end{aligned}$$

Problem 8. Find the inverse of the matrix

$$A = \begin{pmatrix} 2 & -1 & 2 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$$

if it exists.

Problem 9. Assume that

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 3 & -1 & 0 \\ 10 & 0 & -1 \end{pmatrix}.$$

Compute A^2. Is A invertible? If so, find the inverse of A without computing cofactors.