FORK1003
Solutions for Exercises 3

August 6, 2015

1 Introduction to Determinants

Solution 1.1.

(a) 12 (b) 26 (c) 0

2 Clever Trick for 3×3 Determinants

Solution 2.1.

(a) 174 (b) 0 (c) -24

3 Cofactor Expansion

3.1 Minors and Cofactors

Solution 3.1.

(a) $\begin{bmatrix} 0 & 3 & 1 \\ 2 & 0 & 6 \\ -2 & -5 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & -5 & 4 \\ -1 & 2 & 6 \\ 3 & -2 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} -2 & 0 & 3 \\ -1 & 2 & 0 \\ 3 & -2 & -5 \end{bmatrix}$

Solution 3.2.

(a) -8
(b) 26
(c) 30
(d) 4

3.2 Cofactor Expansion

Solution 3.3.

(a) 30

(b) 25

(c) 30 (3rd column)

(d) -496 (4th row)

4 Determinants by Row Reduction

4.1 Determinants and Elementary Row Operations

Solution 4.1.

(a) $-|A|$

(b) $-2|A|$

(c) $|A|$

(d) $|A|$

(e) $2|A|$

(f) $2|A|$

Solution 4.2.

(a) -78. Possible sequence of row operations:

$$\begin{aligned} R1 &\to \frac{1}{2}R1 \\ R2 &\to R2 - R1 \\ R3 &\leftrightarrow R2 \\ R3 &\to R3 + 2R2 \end{aligned}$$

(b) 13. Possible sequence of row operations:

$$\begin{aligned} R1 &\leftrightarrow R2 \\ R1 &\to -R1 \\ R2 &\to R2 + 5R1 \\ R3 &\to R3 - 2R1 \\ R3 &\to R3 + R2 \end{aligned}$$

(c) 256. Possible sequence of row operations:

$$\begin{aligned} R2 &\to R2 - R1 \\ R3 &\to R3 - 2R1 \\ R4 &\to R4 - 6R1 \\ R3 &\to R3 + R2 \\ R4 &\to R4 + 8R2 \\ R4 &\to R4 - \frac{5}{4}R4 \end{aligned}$$

(d) -20. Possible sequence of row operations:

$$\begin{aligned}
&R1 \leftrightarrow R4\\
&R2 \leftrightarrow R3\\
&R3 \to R3 - 4R1\\
&R4 \to R4 - 3R1\\
&R3 \to R3 - R4\\
&R4 \to R4 + 10R2\\
&R3 \to -\frac{1}{5}R3\\
&R4 \to R4 - 12R3
\end{aligned}$$

Solution 4.3.

(a) -72. Start by expanding 3rd row.

(b) 336. Start by the row operation $R5 \to R5 + R1$, followed by expanding the 1st column.

Solution 4.4.

(a) $\det(A) = 9$. $\quad A^{-1} = \frac{1}{9}\begin{bmatrix} 2 & -3 & -2 \\ 4 & 12 & 5 \\ 3 & 9 & 6 \end{bmatrix}$

(b) $\det(B) = -23$. $\quad A^{-1} = -\frac{1}{23}\begin{bmatrix} -4 & 3 & -1 \\ 2 & -13 & 12 \\ 3 & -8 & 18 \end{bmatrix}$

Solution 4.5.

(a)

$$|A| = -9,\ |A_1| = 6,\ |A_2| = -3,\ |A_3| = 0.$$
$$(x_1, x_2, x_3) = (-2/3, 1/3, 0)$$

(b)

$$|A| = -38,\ |A_1| = 38,\ |A_2| = -38,\ |A_3| = -38.$$
$$(x_1, x_2, x_3) = (-1, 1, 1)$$

