

① Lin. Alg.

$$2 \times 2: \begin{aligned} x + y &= 3 \\ x - y &= -1 \end{aligned}$$

$$1: \left[\begin{array}{cc|c} \overset{x}{1} & \overset{y}{1} & 3 \\ 1 & -1 & -1 \end{array} \right]$$

Take $R_2 \mapsto R_2 - R_1$

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 1 & -1 & -1 \end{array} \right] = \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & -2 & -4 \end{array} \right]$$

$(-2y = -4)$

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 1 & \frac{-4}{-2} = 2 \end{array} \right]$$

$R_2 \mapsto \frac{-1}{2} R_2$

$$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array} \right]$$

$R_1 \mapsto R_1 - R_2$

$$\begin{aligned} x + 0y &= 1 \\ 0x + y &= 2 \end{aligned} \rightarrow \begin{cases} x = 1 \\ y = 2 \end{cases}$$

$$\begin{aligned}x_1 + 2x_2 - x_3 &= 2 \\ 2x_1 - x_2 + 5x_3 &= 15 \\ x_1 + 0x_2 + 3x_3 &= 10\end{aligned}$$

~~1st~~

1st column,
1st row

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ \underline{2} & -1 & 5 & 15 \\ \underline{1} & 0 & 3 & 10 \end{array} \right]$$

↓

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & b_1 \\ 0 & 1 & 0 & b_2 \\ 0 & 0 & 1 & b_3 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 2-2 \cdot 1 & -1-2 \cdot 2 & 5+2 & 15-2 \cdot 2 \\ 1-1 & 0-2 & 3+1 & 10-2 \end{array} \right]$$

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$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -5 & 7 & 11 \\ 0 & -2 & 4 & 8 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -2 & 4 & 8 \\ 0 & -5 & 7 & 11 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 1 & -2 & -4 \\ 0 & -5 & 7 & 11 \end{array} \right]$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$R_3 \rightarrow R_3 - R_1$$

$$R_2 \leftrightarrow R_3$$

$$R_2 \rightarrow -\frac{1}{2} R_2$$

②

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 8 \\ 0 & 1 & -2 & -4 \\ 0 & -5 & 7 & -10 \end{array} \right] \begin{array}{l} 2-2=0 \quad -1+4=3 \quad 2+8=10 \\ 0 \\ 0 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 10 \\ 0 & 1 & -2 & -4 \\ 0 & 0 & -3 & -9 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 10 \\ 0 & 1 & -2 & -4 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 10 \\ 0 & 1 & -2 & -4 \\ 0 & 0 & 1 & 3 \end{array} \right] \begin{array}{l} 10-9=1 \\ -4+6=2 \\ 3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right] \rightarrow$$

$$R1 \rightarrow R1 - 2R2$$

$$R3 \rightarrow R3 + 5R2$$

$$R3 \rightarrow -\frac{1}{3}R3$$

$$R1 \rightarrow R1 - 3R3$$

$$R2 \rightarrow R2 + 2R3$$

$$x_1 = 1$$

$$x_2 = 2$$

$$x_3 = 3$$

Pivot = basic

$$\begin{array}{c|ccc} x_1 & x_2 & x_3 & b \\ \hline 1 & 3 & -12 & 0 \\ 0 & 1 & 8 & 13 \\ 0 & 0 & -1 & 1 \end{array}$$

~~free~~

$$\begin{array}{c|ccc} x_1 & x_2 & x_3 & b \\ \hline 1 & 3 & -12 & 0 \\ 0 & 1 & 8 & 13 \\ 0 & 0 & 0 & 1 \end{array}$$

basic ~~free~~ basic

$$\begin{array}{c|ccc} x_1 & x_2 & x_3 & \\ \hline 1 & 3 & -12 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}$$

↓
No sol'n.
 $0x_1 + \dots + 0x_3 = 1$.

3

Pivot
positions

$$\begin{array}{c|cccc} & C_1 & C_2 & C_3 & C_4 \\ \hline 1 & 1 & 5 & 4 & -9 & -7 \\ 2 & 0 & 2 & 1 & -3 & -3 \\ 3 & 0 & 0 & 0 & 1 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \end{array}$$

Pivot columns

x_1 x_2 x_3 x_4

x_3 is free

basic variables

3 x 3 system:

$$\begin{cases} 3x_1 + 2x_2 = -3 \\ x_1 - x_2 + x_3 = 0 \\ -2x_1 - 3x_2 + 2x_3 = 4 \end{cases}$$

Convert to augmented matrix:

$$\left[\begin{array}{ccc|c} 3 & 2 & 0 & -3 \\ 1 & -1 & 1 & 0 \\ -2 & -3 & 2 & 4 \end{array} \right]$$

Reduce to Echelon form

$$\left[\begin{array}{ccc|c} \underline{1} & -1 & 1 & 0 \\ 3 & 2 & 0 & -3 \\ -2 & -3 & 2 & 4 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} \underline{1} & -1 & 1 & 0 \\ 0 & 5 & -3 & -3 \\ 0 & \underline{5} & 4 & 4 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} \underline{1} & -1 & 1 & 0 \\ 0 & \underline{5} & -3 & -3 \\ 0 & 0 & \underline{1} & 1 \end{array} \right]$$

← All basic variables
Unique solution.

$R_1 \leftrightarrow R_2$

$R_2 \rightarrow R_2 - 3R_1$

$R_3 \rightarrow R_3 + 2R_1$

$R_3 \rightarrow R_2 + R_3$