

Problems for Lecture 5

1. Compute the integrals

a) $\int x^2 \ln x \, dx$

b) $\int \frac{x^3 + 2x^2 + 1}{x} \, dx$

c) $\int x e^{-x^2} \, dx$

d) $\int \frac{x}{x^2 + 1} \, dx$

e) $\int \frac{1}{(2x-3)^2} \, dx$

f) $\int x^3 e^{-x^2} \, dx$

g) $\int e^{\sqrt{x}} \, dx$

2. Simplify the expressions using polynomial division:

a) $\frac{x^3}{x^2 - 1}$

c) $\frac{x^3 + x^2 + x + 1}{x + 1}$

b) $\frac{x^2 + 2x - 3}{x + 1}$

d) $\frac{x^4 + 1}{x - 1}$

3. Compute the integrals:

a) $\int \frac{3}{2x-4} \, dx$

d) $\int \frac{x^2}{x^2 - 1} \, dx$

b) $\int \frac{1}{x^2 + x} \, dx$

e) $\int \frac{1}{x^2 - 4x + 4} \, dx$

c) $\int \frac{x}{x^2 - 4x - 5} \, dx$

4. Find the first order partial derivatives and the Hessian:

a) $f(x,y) = x^3 - 3xy + y^3$

b) $f(x,y) = e^{xy}$

c) $f(x,y) = e^{x+2y}$

d) $f(x,y) = \sqrt{x^2 + y^2 + 1}$

e) $f(x,y) = \ln(x^2 + y^2 + 4)$

f) $f(x,y) = \ln(xy) - 1$

g) $f(x,y) = x \ln y - y \ln x$