The first order leading principal minor is $F_{xx} = 6x$ and the second order leading principal minor is $\det D^2F(\mathbf{x}) = -36xy - 81$. At $(0,0)$, these two minors are 0 and -81, respectively. Since the second order leading principal minor is negative, $(0,0)$ is a saddle of F—neither a max nor a min. At $(3,-3)$, these two minors are 18 and 243. Since these two numbers are positive, $D^2F(3,-3)$ is positive definite and $(3,-3)$ is a strict local min of F.

Notice that $(3,-3)$ is not a *global* min, because at the point $(0,n)$, $F(0,n) = -n^3$, which goes to $-\infty$ as $n \to \infty$.

EXERCISES

17.1 For each of the following functions defined on $\mathbf{R}^2$, find the critical points and classify these as local max, local min, saddle point, or "can't tell":

a) $x^4 + x^2 - 6xy + 3y^2$, *b)* $x^2 - 6xy + 2y^2 + 10x + 2y - 5$,
c) $xy^2 + x^3y - xy$, *d)* $3x^4 + 3x^2y - y^3$.

17.2 For each of the following functions defined on $\mathbf{R}^3$, find the critical points and classify them as local max, local min, saddle point, or "can't tell":
a) $x^2 + 6xy + y^2 - 3yz + 4z^2 - 10x - 5y - 21z$,
b) $(x^2 + 2y^2 + 3z^2)e^{-(x^2+y^2+z^2)}$.

17.4 GLOBAL MAXIMA AND MINIMA

The first and second order sufficient conditions of the last section will find all the local maxima and minima of a differentiable function whose domain is an open set in $\mathbf{R}^n$. As Example 17.2 illustrates, these conditions say nothing about whether or not any of these local extrema is a *global* max or min. In this section, we will discuss sufficient conditions for global maxima and minima of a real-valued function on $\mathbf{R}^n$.

The study of one-dimensional optimization problems in Section 3.5 put forth two conditions for a critical point x^* of f to be a global max (or min), when f is a C^2 function defined on a connected interval I of $\mathbf{R}^1$:

(1) x^* is a local max (or min) and it's the only critical point of f in I; or
(2) $f'' \leq 0$ on all of I (or $f'' \geq 0$ on I for a min), that is, f is a concave function on I (or f is a convex function for a min).

Condition 1 does not work in higher dimensions, as the function F whose level sets are pictured in Figure 17.1 illustrates. The point A in Figure 17.1 is a local max of F in the open set U. Even though A is the only critical point of F in U, the function F takes on a higher value at point B.