NORWEGIAN BUSINESS SCHOOL

Department of Economics

Written examination in:	**GRA 60353 Mathematics**
Examination date:	12.12.2011, 09:00 – 12:00
Permitted examination aids:	Bilingual dictionary BI-approved exam calculator: TEXAS INSTRUMENTS BA II Plus™
Answer sheets:	Squares
Total number of pages:	2

Question 1.

We consider the function f given by $f(x, y, z) = x + y + z - \ln(x + 2y + 3z)$ defined on the set $D_f = \{(x, y, z) : x + 2y + 3z > 0\}$.

(a) Find all stationary points of f.
(b) Is f convex? Is it concave?

Question 2.

We consider the matrices A and B, given by

$$A = \begin{pmatrix} 3 & 4 & 5 \\ 0 & 2 & 0 \\ 1 & 3 & 7 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 & 5 \\ 1 & 3 & 5 \\ 1 & 7 & 4 \end{pmatrix}$$

(a) Find all eigenvalues of A, and use them to compute $\det(A)$ and $\operatorname{rk} A$.
(b) Compute all eigenvectors for A. Is A diagonalizable?
(c) Determine if there are any common eigenvectors for A and B. Show that if $\mathbf{x}$ is a common eigenvector for A and B, then $\mathbf{x}$ is also an eigenvector for AB.

Question 3.

We consider the differential equation $(x + 1)t\dot{x} + (t + 1)x = 0$ with initial condition $x(1) = 1$.

(a) Show that the differential equation is separable, and use this to find an implicit expression for $x = x(t)$. In other words, find an equation of the form
$$F(x, t) = A$$
that defines $x = x(t)$ implicitly. It is not necessary to solve this equation for x.
(b) Show that the differential equation becomes exact after multiplication with e^{x+t}. Use this to find an implicit expression for $x = x(t)$. In other words, find an equation of the form
$$G(x, t) = B$$
that defines $x = x(t)$ implicitly. It is not necessary to solve this equation for x.

Question 4.

We consider the optimization problem

$$\min\ 2x^2 + y^2 + 3z^2 \text{ subject to } \begin{cases} x - y + 2z & = 3 \\ x + y & = 3 \end{cases}$$

(a) Write down the first order conditions for this optimization problem and show that there is exactly one admissible point that satisfy the first order conditions, the point $(x, y, z) = (2, 1, 1)$.
(b) Use the bordered Hessian at $(x, y, z) = (2, 1, 1)$ to show that this point is a local minimum for $2x^2 + y^2 + 3z^2$ among the admissible points. What is the local minimum value?
(c) Prove that $(x, y, z) = (2, 1, 1)$ solves the above optimization problem with equality constraints. What is the solution of the Kuhn-Tucker problem

$$\min\ 2x^2 + y^2 + 3z^2 \text{ subject to } \begin{cases} x - y + 2z & \geq 3 \\ x + y & \geq 3 \end{cases}$$

with inequality constraints?