Plan:

1. Final exam 11/2017 (continued)
2. Review: Selected topics

How to prepare for the exam:

Relevant material: Lecture notes, Key problems, Differential Equations, Exam problems, Workbook.

Problem Solving: Make a plan, execute it.
If you do not manage to solve, what went wrong?
What can you learn?

- Learn definitions / basics. Lecture notes
- Simple problems are the most important ones. Be realistic and strategic when you choose what you practise.

Remember to do the course evaluation.

① Exam 11/2017, Question 4:

max $f = x^2y^2$ when $x^2+y^2+x^2y^2 \leq 3$

a) $\mathcal{L} = x^2y^2 - \lambda(x^2+y^2+x^2y^2)$

FOC $\begin{cases} \mathcal{L}'_x = 2xy^2 - \lambda(2x+2xy^2) = 2x(y^2-\lambda-\lambda y^2) = 0 \\ \mathcal{L}'_y = 2x^2y - \lambda(2y+2x^2y) = 2y(x^2-\lambda-\lambda x^2) = 0 \end{cases}$

C : $x^2+y^2+x^2y^2 \leq 3$

CSC: $\lambda \cdot (x^2+y^2+x^2y^2-3) = 0$, $\lambda \geq 0$

b) FOC: $x, y \neq 0$

$y^2 - \lambda - \lambda y^2 = 0 \quad y^2(1-\lambda) = \lambda \quad y^2 = \frac{\lambda}{1-\lambda}$ $1-\lambda \neq 0$

$x^2 - \lambda - \lambda x^2 = 0 \quad x^2(1-\lambda) = \lambda \quad x^2 = \frac{\lambda}{1-\lambda}$

$\Downarrow$

$x^2 = y^2$

C: binding case $\begin{cases} x^2+y^2+x^2y^2 = 3 \\ x^2+x^2+x^4 = 3 \\ x^4+2x^2-3=0 \\ \\ x^2=1: \frac{\lambda}{1-\lambda} = 1 \\ \lambda = 1-\lambda \\ 2\lambda = 1 \quad \lambda = 1/2 \end{cases}$

$x^2 = \frac{-2 \pm \sqrt{4-4(-3)}}{2} = \frac{-2+4}{2} = 1, -3$

$x^2 = 1 \quad y^2 = 1 \quad \Rightarrow \quad (\pm 1, \pm 1; \lambda)$

$\lambda \geq 0$ ok

$f = 1 \quad \boxed{(\pm 1, \pm 1; 1/2)}$

non-binding case $\begin{cases} x^2+y^2+x^2y^2 < 3, \ \lambda = 0 \\ y^2 = x^2 = 0 \\ \text{no pts. with } x, y \neq 0 \end{cases}$

Conclusion: Ordinary candidate pts. (sol's of FOC+C+CSC) with $x, y \neq 0$ are:

$(\pm 1, \pm 1; 1/2) \quad f = 1$

c) Show that the problem has a maximum

EVT (extreme value thm.) : adm pts. bounded set ⟹ there is a maximum

SOC (second order cond.)

$h(x,y) = \mathcal{L}(x,y;\lambda^*) = h(x,y;\frac{1}{2})$ concave

⇓

(x^*,y^*) is maximum

Extreme value thm:

$x^2+y^2+x^2y^2 \le 3$ is bounded ⟹ there is a maximum

EVT

because

$-\sqrt{3} \le x \le \sqrt{3}$

$-\sqrt{3} \le y \le \sqrt{3}$

Find the maximum value:

Possible maxim pts:

Ordinary candidate pts (FOC + C + CSC)

$xy \neq 0$: $(\pm 1, \pm 1; 1/2)$ $f = 1$

$x = 0$ or $y = 0$: $\left.\begin{array}{l}(0,y;\lambda)\\(x,0;\lambda)\end{array}\right\}$ $f = 0$

Adm. pts. where NDCQ fails: — no points

Since these are all candidate pts, and the problem has a maximum, $\underline{\underline{f_{max} = 1}}$ at $(x,y) = (\pm 1, \pm 1)$

NDCQ: $x^2+y^2+x^2y^2 \le 3$

a) $x^2+y^2+x^2y^2=3$: NDCQ: $\mathrm{rk}\,(\underbrace{2x+2xy^2}_{g'_x} \quad \underbrace{2y+2yx^2}_{g'_y}) = 1$

NDCQ fails: $\mathrm{rk}\,(g'_x \quad g'_y) < 1$

$\mathrm{rk}\,(2x+2xy^2 \quad 2y+2yx^2) = 0$

$2x+2xy^2=0$ and $2y+2yx^2=0$

$2x(1+y^2)=0$ $\quad$ $2y(1+x^2)=0$

$x=0$ or $\cancel{1+y^2=0}$, $\cancel{y^2=-1}$ $\quad$ $y=0$ or $\cancel{1+x^2=0}$, $\cancel{x^2=-1}$

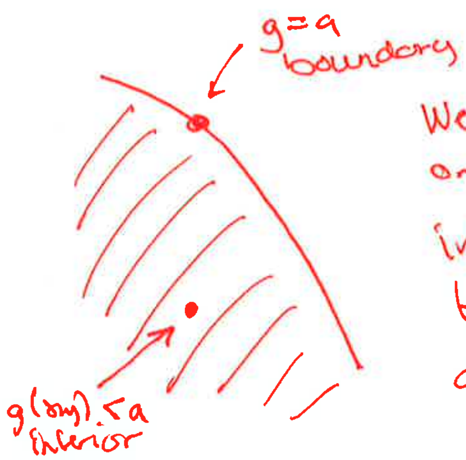
$\Downarrow$

$(x,y)=(0,0)$ but this point does not satisfy $x^2+y^2+x^2y^2=3$

no points in this case

b) $x^2+y^2+x^2y^2<3$: NDCQ: no condition

no points in this case



We only consider NDCQ on the boundary, in the interior it is not necessary to check anything, NDCQ always satisfied at interior pts.

Exam	Final exam in GRA 6035 Mathematics
Date	November 29th, 2017 at 0900 - 1200

This exam consists of 12+1 problems (one additional problem is for extra credits, and can be skipped). Each problem has a maximal score of 6p, and 72p (12 solved problems) is marked as 100% score.

You must give reasons for your answers. Precision and clarity will be emphasized when evaluating your answers.

QUESTION 1.

We consider the quadratic form f given by $f(x, y, z, w) = x^2 + y^2 + 9z^2 + 4w^2 + 6yz - 4yw - 12zw$.

(a) **(6p)** Find the symmetric matrix A of the quadratic form f, and compute the rank of A.
(b) **(6p)** Determine the definiteness of the quadratic form f.
(c) **(6p)** Find two vectors $\mathbf{v}_1, \mathbf{v}_2$ such that $\text{span}(\mathbf{v}_1, \mathbf{v}_2)$ is the set of solutions of $A\mathbf{x} = \mathbf{0}$.

QUESTION 2.

Find the general solutions of the following differential equations:

(a) **(6p)** $y' - 2y = e^t$
(b) **(6p)** $3t^2 - y - ty' = 0$

Find all equilibrium states of the following differential equation, and determine their stability. Are any of the equilibrium states globally asymptotically stable?

(c) **(6p)** $y' = 2y(3 - y)$

QUESTION 3.

Let u be the function given by $u(x, y, z) = 1 + x^2 + 5y^2 + 8z^2 + 4xy - 2yz$, and consider the composite function $f(x, y, z) = \ln(u)/u^2$ with $u = u(x, y, z)$.

(a) **(6p)** Find the minimal value of $u = u(x, y, z)$, if it exists.
(b) **(6p)** Compute the first order partial derivatives of $f = f(x, y, z)$.
(c) **(6p)** Determine the maximum and minimum values of f.

QUESTION 4.

We consider the following Kuhn-Tucker problem:

$$\max\ f(x, y) = x^2y^2 \text{ subject to } x^2 + y^2 + x^2y^2 \leq 3$$

(a) **(6p)** Write down all Kuhn-Tucker conditions for this problem.
(b) **(6p)** Find all points $(x, y; \lambda)$ with $x, y \neq 0$ that satisfy the Kuhn-Tucker conditions.
(c) **(6p)** Show that the Kuhn-Tucker problem has a maximum, and find the maximum value.

QUESTION 5.

Extra credit (6p) Find the general solution of following system of differential equations:

$$\begin{pmatrix} y' \\ z' \end{pmatrix} = \begin{pmatrix} 5 & -6 \\ 1 & -2 \end{pmatrix} \cdot \begin{pmatrix} y \\ z \end{pmatrix}$$

2 Review:

1. Matrix methods
2. Differential equations
   - separable
   - linear first order ✓
   - exact ✓
   - linear sec. order
   - systems
   - Eq. states and stability

Exam 12/2016 Q 2b.

$$y' - y \cdot \ln t = y$$

linear first order $y' + a(t)y = b(t)$
int. factor

$$y' - y\ln t - y = 0$$

$$a(t) = -\ln t - 1 = -(\ln t + 1)$$

$$y' - (\ln t + 1)y = 0$$

$$\int a(t)\,dt = \int \underbrace{-1}_{u'} \cdot \underbrace{(\ln t + 1)}_{v}\,dt$$

$$u = -t \qquad v' = 1/t$$

$$= -t \cdot (\ln t + 1) - \int -\cancel{t} \cdot \frac{1}{\cancel{t}}\,dt$$

$$= -t\ln t - t + t + C = -t\ln t + C$$

$$\Downarrow$$

$$u = e^{-t\ln t} \quad \text{int. factor} \quad (C = 0)$$

$$(y \cdot e^{-t\ln t})' = 0 \cdot e^{-t\ln t} = 0$$

$$y e^{-t\ln t} = C$$

$$y = C \cdot e^{t\ln t}$$

Exam 06/2017, Q 2c.

$$\frac{y-2t}{ty-t^2} + \frac{t}{ty-t^2}\, y' = 1 \Rightarrow \underbrace{\frac{y-2t}{ty-t^2} - 1}_{p} + \underbrace{\frac{t}{ty-t^2}}_{q}\, y' = 0$$

Exact?

$$h'_t = p = \frac{y-2t}{ty-t^2} - 1$$

$$h'_y = q = \frac{t}{ty-t^2}$$

$$h = \int \frac{y-2t}{ty-t^2} - 1 \, dt = \int \frac{y-2t}{ty-t^2}\, dt - t$$

$$= \int \frac{\cancel{y-2t}}{u} \cdot \frac{du}{\cancel{y-2t}} - t = \ln|ty-t^2| - t + g(y)$$

$u = ty - t^2$

$du = (y-2t)\, dt$

$$h'_y = \frac{t}{ty-t^2} + g'(y) = \frac{t}{ty-t^2}$$

We take $g(y) = 0$

$g'(y) = 0$

the eqn. is exact

Solution: $h = \ln|ty-t^2| - t = C$

$$\ln|ty-t^2| = C + t$$

$$|ty-t^2| = e^{C+t}$$

$$ty - t^2 = (\pm e^C)\, e^t = Ke^t$$

$$\frac{ty}{t} = \frac{t^2 + Ke^t}{t}$$

$$y = \frac{t^2 + Ke^t}{t}$$

3 - 4 Optimization (Unconstrained / constrained)

	Unconstr.	Lagr.	KT.
Candidate pts:	FOC	FOC+C	FOC+C+CSC
Max/min:	Convex/concave; H(f) is pos./neg. semidefin. at all pts.	SOC or EVT+NDCQ	SOC or EVT+NDCQ
Envelope thm	$df^*(a)/da = \frac{\partial f}{\partial a}(x^*(a),\dots)$	$df^*(a)/da = \frac{\partial L}{\partial a}(x^*(a),\dots,\lambda^*(a),\dots)$	

Exam 12/2016, Q 4c. min $f = 5x^2 - 8xy \dots$ when $x+y-4z=8$

a) b): $f^* = 33$ at $x^* = 2z+4$ $y^* = 2z+4$ $z^* = z$ $\lambda^* = 8$

c) Problem with parameter a:

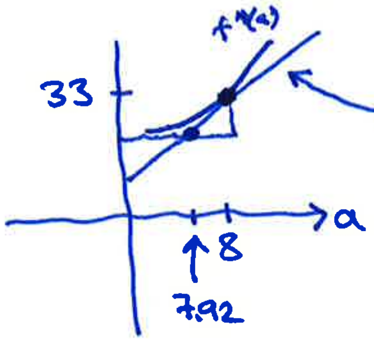
min $f(x,y,z)$ when $x+y-4z=a \rightarrow x+y-4z-a=0$

$$L = f(x,y,z) - \lambda\cdot(x+y-4z-a) = \dots + \lambda a$$

$$L'_a = \lambda$$

$f^*(8) = 33$

$(x^*(8), y^*(8), z^*(8); \lambda^*(8)) = (2z+4, 2z+4, z; 8)$



Envelope Thm: $\frac{df^*(a)}{da} = L'_a(x^*(a), y^*(a), z^*(a); \lambda^*(a)) = \lambda^*(a) = 8$ at $a=8$

slope of tangent line

$$f^*(7.92) \simeq \underset{33}{f^*(8)} + \underset{-0.08}{\Delta a}\cdot \underset{8}{\frac{df^*(a)}{da}} = 32.36$$

new minimum value when $a = 7.92$

Exam 12/2014 Q4

max $f = x + 4y + 2z + 5w$ when $2x^2 + 2y^2 + 2yz + 2z^2 + 2w^2 \leq 21$

a) $L = x + 4y + 2z + 5w - \lambda(2x^2 + 2y^2 + 2yz + 2z^2 + 2w^2)$

FOC: $L'_x = 1 - \lambda \cdot 4x = 0$
$L'_y = 4 - \lambda \cdot (4y + 2z) = 0$
$L'_z = 2 - \lambda \cdot (2y + 4z) = 0$
$L'_w = 5 - \lambda \cdot 4w = 0$

C: $2x^2 + 2y^2 + 2yz + 2z^2 + 2w^2 \leq 21$

CSC: $\lambda \geq 0$ and
$\lambda(2x^2 + 2y^2 + 2yz + 2z^2 + 2w^2 - 21) = 0$

i) Non-binding case: $g(x,y,z,w) < 21$ C
$\lambda = 0$
$1 - 0 \cdot 4x = 0$ impossible $\Rightarrow$ no points in this case

ii) Binding case: $g(x, \ldots, w) = 21$ C
$\lambda \geq 0$ CSC

$1 - 4\lambda x = 0$
$5 - 4\lambda w = 0$ $\Rightarrow$ $\boxed{x = \frac{1}{4\lambda} \quad y = 1/\lambda \quad w = \frac{5}{4\lambda} \quad z = 0}$

$4 = \lambda \cdot (4y + 2z) \Rightarrow 4y + 2z = \frac{4}{\lambda}$
$2 = \lambda \cdot (2y + 4z) \Rightarrow 2y + 4z = \frac{2}{\lambda}$
$\uparrow$
$z = 0$

$2y = \frac{2}{\lambda}$
$y = \frac{1}{\lambda}$

$\frac{4}{\lambda} = 2 \cdot \frac{2}{\lambda}$
$(4y + 2z) = 2 \cdot (2y + 4z)$
$\cancel{4y} + 2z = \cancel{4y} + 8z$
$\frac{0}{6} = \frac{6z}{6}$
$z = 0$

C: $2x^2+2y^2+2yz+2z^2+2w^2=21$

$x = 1/4\lambda$
$y = 1/\lambda = 4/4\lambda$
$z = 0$
$w = 5/4\lambda$

$$2\left(\frac{1}{4\lambda}\right)^2 + 2\cdot\left(\frac{4}{4\lambda}\right)^2 + 2\cdot\left(\frac{5}{4\lambda}\right)^2 = 21$$

$$\left(\frac{1}{4\lambda}\right)^2 \cdot \left(\underbrace{2\cdot 1^2 + 2\cdot 4^2 + 2\cdot 5^2}_{2+32+50}\right) = 21$$

$$\left(\frac{1}{4\lambda}\right)^2 \cdot 84 = 21$$

$$\left(\frac{1}{4\lambda}\right)^2 = \frac{21}{84} = 1/4$$

$$\frac{1}{4\lambda} = \pm\sqrt{1/4} = \pm 1/2 \longleftarrow \boxed{\text{CSC: } \lambda \geq 0}$$

$$\frac{1}{4\lambda} = \frac{1}{2} \quad | \cdot 4\lambda$$

$$1 = 2\lambda$$

$$\underline{\lambda = 1/2} \qquad (x,y,z,w) = \left(\tfrac{1}{2}, 2, 0, \tfrac{5}{2}\right)$$

Conclusion: FOC+C+CSC gives $\underline{(x,y,z,w;\lambda) = \left(\tfrac{1}{2}, 2, 0, \tfrac{5}{2}; 1/2\right)}$

$f = x + 4y + 2z + 5w$

$f = \frac{1}{2} + 8 + \frac{25}{2}$
$\underline{= 21}$

SOC for:

b) $h(x,y,z,w) = f(x,y,z,w) - \frac{1}{2}(2x^2+2y^2+2yz+2z^2+2w^2)$

$$H(h) = \begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & -1 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$$

$D_1 = -2$
$D_2 = 4$
$D_3 = -2\cdot(4-1) = -6$
$D_4 = -2(-6) = 12$

H(h) is neg. defin. (and therefore also neg. semidef.)
$\Downarrow$
h concave $\xRightarrow{\text{SOC}}$ $(1/2, 2, 0, 5/2)$ is max and

$f_{max} = f(1/2, 2, 0, 5/2) = \underline{\underline{21}}$

c) max $f = x + ay + 2z + bw$ when $g(x,y,z,w) \le 21$

$L = x + ay + 2z + bw - \lambda(g(x,y,z,w) - 21)$

$f^*(a,b) =$ max. value for param. a, b.

$f^*(4,5) = 21$ at $(\frac{1}{2}, 2, 0, \frac{5}{2}; \frac{1}{2}) = (x^*(4,5), y^*(4,5), \ldots)$

Max. value when $a = 3.8$ and $b = 5.4$:

$$f^*(3.8, 5.4) \simeq \underset{\substack{\| \\ 21}}{f^*(4,5)} + \underset{\substack{\| \\ -0.2}}{\Delta a} \cdot \underset{\substack{\| \\ 2}}{\frac{\partial f^*(a,b)}{\partial a}} + \underset{\substack{\| \\ 0.4}}{\Delta b} \cdot \underset{\substack{\| \\ 2.5}}{\frac{\partial f^*(a,b)}{\partial b}}$$

$$= 21 - 0.4 + 1 = \underline{\underline{21.6}}$$

$L'_a = y \Rightarrow \dfrac{\partial f^*(a,b)}{\partial a} = y^*(a,b) = 2$

$L'_b = w \Rightarrow \dfrac{\partial f^*(a,b)}{\partial b} = w^*(a,b) = 5/2$

from the envelope thm.

Exam	Final exam in GRA 6035 Mathematics
Date	December 1st, 2016 at 0900 - 1200

All subquestions have the same weight and give maximal score 6p each. Answers to the first 12 subquestions give a maximal score of 72p (100%). Question 5 can be skipped, but gives 6p extra credit if answered correctly.

Question 1.

We consider the matrix A given by

$$A = \begin{pmatrix} -2 & 2 & 0 \\ -1 & 0 & 2 \\ 0 & -1 & 2 \end{pmatrix}$$

(a) **(6p)** Compute the determinant and rank of A.
(b) **(6p)** Solve the linear system $A \cdot \mathbf{x} = \mathbf{0}$, and write the solutions in the form $\text{span}(\mathbf{v}_1, \ldots, \mathbf{v}_r)$.
(c) **(6p)** Find all eigenvalues of A and their multiplicities.

Question 2.

Solve the difference equation:

(a) **(6p)** $y_{t+2} = 3y_{t+1} - 2y_t, \; y_0 = 1, \; y_1 = 2$

Solve the differential equations:

(b) **(6p)** $y' - y \ln t = y$
(c) **(6p)** $ye^{ty} + te^{ty}y' = 1, \; y(1) = \ln 2$

Question 3.

We consider the function given by $f(x, y, z) = 5x^2 - 8xy - 4xz + 5y^2 - 4yz + 8z^2 + 1$.

(a) **(6p)** Is f convex?
(b) **(6p)** Find all the stationary points of f.
(c) **(6p)** Find the minimum value of $g(x, y, z) = w \ln(w)$, with $w = f(x, y, z)$, if it exists.

Question 4.

We consider the following Lagrange problem:

$$\min \; f(x, y, z) = 5x^2 - 8xy - 4xz + 5y^2 - 4yz + 8z^2 + 1 \text{ subject to } x + y - 4z = 8$$

(a) **(6p)** Write the Lagrange conditions as a linear system and find its augmented matrix.
(b) **(6p)** Solve the Lagrange problem. What is the minimum value?
(c) **(6p)** Consider the new Lagrange problem where the constraint is replaced by $x+y-4z = 7.92$. State the relevant envelope theorem, and use it to estimate the new minimum value.

Question 5.

Let $\alpha_1, \alpha_2, \alpha_3$ be parameters and consider the matrix A given by

$$A = \begin{pmatrix} -\alpha_2 & \alpha_1 & 0 \\ -\alpha_3 & 0 & \alpha_1 \\ 0 & -\alpha_3 & \alpha_2 \end{pmatrix}$$

Extra credits (6p)
Compute the rank of A for all values of $(\alpha_1, \alpha_2, \alpha_3)$.

Exam	Final exam in GRA 6035 Mathematics
Date	June 20th, 2017 at 0900 - 1200

This exam consists of 12+1 problems (one additional problem is for extra credits, and can be skipped). Each problem has a maximal score of 6p, and 72p (12 solved problems) is marked as 100% score.

You must give reasons for your answers. Precision and clarity will be emphasized when evaluating your answers.

Question 1.

We consider the matrix A given by

$$A = \begin{pmatrix} 1+a & 2 & 2 \\ 2 & 1+a & 2 \\ 2 & 2 & 1+a \end{pmatrix}$$

(a) Compute the determinant and rank of A when $a = -5$.
(b) When $a = -5$, find a vector $\mathbf{v}$ such that $\text{span}(\mathbf{v})$ is the set of solutions of $A \cdot \mathbf{x} = \mathbf{0}$.
(c) Determine all values of a such that $\text{rk}\, A = 2$.
(d) Find a diagonal matrix D such that $D = P^{-1}AP$ for an invertible matrix P.

Question 2.

We consider differential equations in the function $y = y(t)$.

(a) Solve the differential equation $y'' + y' - 6y = 36t$.
(b) Solve the differential equation $ty' - y = \ln(t)$.
(c) Show that the differential equation

$$\frac{y - 2t}{ty - t^2} + \frac{t}{ty - t^2} \cdot y' = 1$$

is both linear and exact, and solve it.

Question 3.

We consider the functions $f(x, y, z) = 9 - x^2 - y^2 - z^2 + 2xz$ and $g(x, y, z) = \ln(10 - f(x, y, z))$.

(a) Explain that f is concave, and find its maximum value.
(b) Find all stationary points of g.
(c) Determine whether g has a maximal and/or a minimal value. Give your answer as the interval of possible values $w = g(x, y, z)$ of g. It is not necessary to compute the Hessian matrix of g.

Question 4.

We consider the following Kuhn-Tucker problem:

$$\max\ f(x, y, z) = 9 - x^2 - y^2 - z^2 + 2xz \quad \text{subject to} \quad x + y - z \geq 2$$

(a) Write down all Kuhn-Tucker conditions for this problem.
(b) Solve the Kuhn-Tucker problem and find its maximum value.

Consider the function $f_a(x, y, z) = 9 - x^2 - ay^2 - z^2 + 2xz$ with parameter a, and the Kuhn-Tucker problem where the objective function f is replaced by f_a.

(c) Explain that the new Kuhn-Tucker problem has a maximum value when $a > 0$, and estimate this maximum value when $a = 1.25$.

Exam	Final exam in GRA 6035 Mathematics
Date	December 12th, 2014 at 0900 - 1200

All subquestions have the same weight and give maximal score 6p each. Answers to the first 12 subquestions give a maximal score of 72p (100%). Question 5 can be skipped, but gives 6p extra credit if answered correctly.

QUESTION 1.

We consider the function given by $f(x, y, z) = x^4 + y^2 - xz + z^4$.

(a) **(6p)** Compute the partial derivatives and the Hessian matrix of f.
(b) **(6p)** Find all stationary points of f, and classify them as local max, local min or saddle points.
(c) **(6p)** Is f convex?

QUESTION 2.

We consider the matrix A given by

$$A = \begin{pmatrix} t & 1 & 1 \\ 1 & t & 1 \\ 1 & 1 & t \end{pmatrix}$$

(a) **(6p)** Compute the determinant of A, and the rank of $A - \lambda I$ when $\lambda = t - 1$.
(b) **(6p)** Show that A is diagonalizable when $t = 8$, and find all its eigenvalues in this case.

A car rental firm has three locations and 120 cars. We assume that all cars are returned after one week, and that any rented car is 8 times as likely to be returned to the pick-up location as any of the other locations.

(c) **(6p)** Find the transition matrix of the resulting Markov chain. If one of the three locations is at an airport, and 50 of the cars are starting out at this location, how many cars will be at the airport location in the long run?

QUESTION 3.

Solve the difference equation:

(a) **(6p)** $y_{t+1} - 3y_t = -5(2t + 1)$

Solve the differential equations:

(b) **(6p)** $t^3 y' = y^2$
(c) **(6p)** $(2yt - 1)y' = (t + 1)e^t - y^2$

QUESTION 4.

We consider the following Kuhn-Tucker problem:

$$\max\ f(x, y, z, w) = x + 4y + 2z + 5w \text{ subject to } 2x^2 + 2y^2 + 2yz + 2z^2 + 2w^2 \leq 21$$

(a) **(6p)** Write down the Kuhn-Tucker conditions for this problem, and find all points that satisfy these conditions. (You will find that the Lagrange multiplier $\lambda = 1/2$).
(b) **(6p)** Solve the Kuhn-Tucker problem and find the corresponding maximum value.
(c) **(6p)** Use (a) and (b) to estimate the maximum value in the Kuhn-Tucker problem

$$\max\ x + 3.8y + 2z + 5.4w \text{ subject to } 2x^2 + 2y^2 + 2yz + 2z^2 + 2w^2 \leq 21$$