GRA 6035 Mathematics

Problems for Lecture 11

Key problems

Problem 1.
Solve the exact differential equations:
$a)\ 3t^2 - 2t + 2y \cdot y' = 0 \quad b)\ 2y - 3t^2 + 2(y+t)y' = 0 \quad c)\ \dfrac{y(1-2\ln t)}{t^3} + \dfrac{\ln t}{t^2} \cdot y' = 0$

Problem 2.
Find the equilibrium states and determine their stability:
$a)\ y' = 6 - 2y \quad b)\ y' = y^2 - 4 \quad c)\ y' = 5y(1 - y/10)$
In each case, sketch solutions curves in the (t, y)-coordinate system to illustrate the results.

Problem 3.
Solve the differential equations:
$a)\ y'' + 6y' - 16y = 16t - 22 \quad b)\ y'' + 6y' + 9y = 4e^{-t} \quad c)\ y'' - 3y' + 2y = 3e^{2t} \quad d)\ y'' - y = t^2$

Problems from Differential Equations

Exercise problems	1.17 - 1.34 (full solutions on the web page)

Problems from the Digital Workbook

Exercise problems	10.13 - 10.16, 11.1 - 11.16 (full solutions in the workbook)
Excel problems	11.17 (full solutions in the workbook) As a minimum, you should understand what happens when you change the parameters in the Excel models that are available in the workbook.

Answers to key problems

Problem 1.
$a)\ y = \pm\sqrt{t^2 - t^3 + C} \quad b)\ y = -t \pm \sqrt{t^2 + t^3 + C} \quad c)\ y = \dfrac{Ct^2}{\ln t}$

Problem 2.
$a)\ y_e = 3$ is globally asymptotically stable
$b)\ y_e = -2$ is stable (but not globally asymptotically stable), $y_e = 2$ is unstable
$c)\ y_e = 0$ is unstable, $y_e = 10$ is stable (but not globally asymptotically stable)

Problem 3.
$a)\ y = C_1\, e^{-8t} + C_2\, e^{2t} + 1 - t \quad b)\ y = C_1\, e^{-3t} + C_2 t e^{-3t} + e^{-t} \quad c)\ y = C_1\, e^{2t} + C_2 e^{t} + 3te^{2t} \quad d)\ y = C_1\, e^{t} + C_2\, e^{-t} - t^2 - 2$