GRA 6035 Mathematics

Problems for Lecture 2

Key problems

Problem 1.
Compute the determinant of these matrices:

$$a)\quad A=\begin{pmatrix}1&2&5\\3&1&2\\1&2&4\end{pmatrix} \qquad b)\quad A=\begin{pmatrix}1&a&b\\a&1&c\\b&c&1\end{pmatrix} \qquad c)\quad A=\begin{pmatrix}1&0&0&3\\0&4&2&0\\0&2&4&0\\3&0&0&1\end{pmatrix}$$

Problem 2.
Use minors to determine the rank of these matrices. Can you find the pivot positions based on the minors?

$$a)\quad A=\begin{pmatrix}4&1&1&3&7\\2&1&0&1&0\\1&0&3&1&0\end{pmatrix} \qquad b)\quad A=\begin{pmatrix}1&3&2&4\\2&-1&7&3\\4&5&11&10\end{pmatrix} \qquad c)\quad A=\begin{pmatrix}3&0&0&7\\0&5&5&0\\0&5&5&0\end{pmatrix}$$

Problem 3.
Use minors to find the rank of these matrices:

$$a)\quad A=\begin{pmatrix}1&3&t\\2&5&7\end{pmatrix} \qquad b)\quad A=\begin{pmatrix}a&7&-3&5&10\\2&-3&1&4&18\\1&24&-10&11&12\end{pmatrix} \qquad c)\quad A=\begin{pmatrix}1&a&b\\a&b&1\end{pmatrix}$$

Problem 4.
Use minors to determine the number of solutions of these linear systems. What are the possible choices of free variables, if any?

$$a)\quad \begin{aligned}x+\ y+\ z&=6\\ x+2y+tz&=13\\ x+3y+9z&=24\end{aligned} \qquad b)\quad \begin{aligned}x+4y+5z-3w&=6\\ 2x+7y+\ z\ \ \ \ \ \ &=4\\ x+5y+4z-8w&=1\end{aligned}$$

Problems from the Digital Workbook

Exercise problems	2.1 - 2.22 (full solutions in the workbook)
Exam problems	2.23 - 2.25 (full solutions in the workbook)

Answers to key problems

Problem 1.
a) $|A|=5$ b) $|A|=1-a^2-b^2-c^2+2abc$ c) $|A|=-96$

Problem 2.
a) $\operatorname{rk}A=3$ with $A=\begin{pmatrix}4&1&1&3&7\\2&1&0&1&0\\1&0&3&1&0\end{pmatrix}$ b) $\operatorname{rk}A=3$ with $A=\begin{pmatrix}1&3&2&4\\2&-1&7&3\\4&5&11&10\end{pmatrix}$ c) $\operatorname{rk}A=2$ with $A=\begin{pmatrix}3&0&0&7\\0&5&5&0\\0&5&5&0\end{pmatrix}$

Problem 3.
a) $\operatorname{rk}A=2$ for all t b) $\operatorname{rk}A=2$ if $a=1$, and $\operatorname{rk}A=3$ if $a\neq 1$ c) $\operatorname{rk}A=1$ if $(a,b)=(1,1)$ and $\operatorname{rk}A=2$ if $(a,b)\neq(1,1)$

Problem 4.
a) One solution if $t\neq 5$, and no solutions if $t=5$ b) Infinitely many solutions (one degree of freedom).
In b) we may choose x, y, z or w as a free variable.