GRA 6035 MATHEMATICS
Problems for Lecture 5

Key problems

Problem 1.
Find the equilibrium state $\mathbf{v}$ of the Markov chains with transition matrix A:

a) $A = \begin{pmatrix} 0.30 & 0.15 \\ 0.70 & 0.85 \end{pmatrix}$
b) $A = \begin{pmatrix} 0.86 & 0.42 \\ 0.14 & 0.58 \end{pmatrix}$
c) $A = \begin{pmatrix} 0.75 & 0.02 & 0.10 \\ 0.20 & 0.90 & 0.20 \\ 0.05 & 0.08 & 0.70 \end{pmatrix}$
d) $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

Problem 2.
Determine the definiteness of the symmetric matrix:

a) $A = \begin{pmatrix} 7 & 4 \\ 4 & 3 \end{pmatrix}$
b) $A = \begin{pmatrix} -1 & 1 \\ 1 & -3 \end{pmatrix}$
c) $A = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 5 & 0 \\ 1 & 0 & 4 \end{pmatrix}$
d) $A = \begin{pmatrix} 2 & 3 & -5 \\ 3 & 7 & 0 \\ -5 & 0 & 35 \end{pmatrix}$
e) $A = \begin{pmatrix} -1 & -2 & -2 \\ -2 & -4 & -4 \\ -2 & -4 & -2 \end{pmatrix}$

Problem 3.
Find the symmetric matrix of the quadratic form, and determine its definiteness:
a) $f(x,y) = x^2 - 8xy + 3y^2$ b) $f(x,y,z) = 2x^2 - 2xz + 3y^2 + z^2$ c) $f(x,y,z) = 3x^2 + 4xy - 4xz + 3y^2 + 4yz + 8z^2$
d) $f(x,y) = 2xy - y^2$ e) $f(x,y,z,w) = xw - yz$

Problem 4.
Determine the definiteness of the symmetric matrix:
$$A = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$$

Problems from the Digital Workbook

Exercise problems	5.1 - 5.9 (full solutions in the workbook)
Exam problems	5.10 - 5.15 (full solutions in the workbook)
	Midterm exam 01/2018 Question 1-6, 8
	Midterm exam 05/2018 Question 1-6, 8

Answers to key problems

Problem 1.
a) $\mathbf{v} = \begin{pmatrix} 3/17 & 14/17 \end{pmatrix}^T$ b) $\mathbf{v} = \begin{pmatrix} 3/4 & 1/4 \end{pmatrix}^T$ c) $\mathbf{v} = \begin{pmatrix} 2/15 & 10/15 & 3/15 \end{pmatrix}^T$ d) No equilibrium unless $\mathbf{v}_0 = \begin{pmatrix} a & b & a \end{pmatrix}^T$

Problem 2.
a) Positive definite   b) Negative definite   c) Positive definite   d) Positive semi-definite   e) Indefinite

Problem 3.
a) Indefinite   b) Positive definite   c) Positive semi-definite   d) Indefinite   e) Indefinite

Problem 4.
Positive semidefinite